

PEMBAHASAN DAN KUNCI JAWABAN

BPM BUKU
PENDALAMAN
MATERI



USBN

SMA/MA

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MATEMATIKA IPA

PEMBAHASAN DAN KUNCI JAWABAN LATIHAN SOAL

MATEMATIKA IPA

Bab 1 Logika Matematika

- Berdasarkan modus tollens,
Hari hujan $\Rightarrow$ ibu memakai payung
 $\sim$ (ibu memakai payung)
 $\therefore \sim$ (hari hujan)
Jadi, kesimpulan yang sah adalah hari tidak hujan.
Jawaban: A
- Berdasarkan silogisme dan modus tollens,
Badu rajin bekerja $\Rightarrow$ ia disayangi ibu
Badu disayangi ibu $\Rightarrow$ ia disayangi nenek
Badu rajin bekerja $\Rightarrow$ ia disayangi nenek
 $\sim$ (Badu disayangi nenek)
 $\therefore \sim$ (Badu rajin bekerja)
Jadi, kesimpulan yang sah adalah Badu tidak rajin bekerja.
Jawaban: E
- Pernyataan, "Jika hari hujan, maka sungai meluap".
 $p \Rightarrow q \equiv \sim q \Rightarrow \sim p \equiv \sim p \vee q$
Sehingga pernyataan di atas ekuivalen dengan, "Jika sungai tidak meluap, maka hari tidak hujan" atau "Hari tidak hujan atau sungai meluap".
Jawaban: D
- Pernyataan, "Semua orang makan nasi".
 $\sim$ (Semua p) $\equiv$ Ada/beberapa $\sim p$
Sehingga ingkaran pernyataan di atas adalah "Beberapa orang tidak makan nasi".
Jawaban: A
- $\sim (p \Rightarrow q) \equiv p \wedge \sim q$ sehingga
 $\sim [(p \wedge q) \Rightarrow r] \equiv (p \wedge q) \wedge \sim r$ atau $p \wedge q \wedge \sim r$
Jawaban: C

Bab 2 Pangkat, Akar, dan Logaritma

$$1. \frac{8a^2b^3c^{-1}}{64a^{-2}bc^2} = \frac{8}{64} a^{2-(-2)} b^{3-1} c^{-1-2}$$

$$= \frac{1}{8} a^4 b^2 c^{-3} = \frac{a^4 b^2}{8c^3}$$

Jawaban: B

$$2. \frac{\left(7^{\frac{4}{5}} \cdot 625^{\frac{1}{8}}\right)}{\left(125^{-\frac{1}{2}} \cdot 49^{\frac{3}{5}}\right)} = \frac{7^{\frac{4}{5}} \cdot (5^4)^{\frac{1}{8}}}{(5^3)^{-\frac{1}{2}} \cdot (7^2)^{\frac{3}{5}}} = \frac{7^{\frac{4}{5}} \cdot 5^{\frac{1}{2}}}{5^{-\frac{3}{2}} \cdot 7^{\frac{6}{5}}} = 5^{\frac{1}{2} - (-\frac{3}{2})} \cdot 7^{\frac{4}{5} - \frac{6}{5}}$$

$$= 5^{\frac{4}{2}} \cdot 7^{-\frac{10}{5}} = 5^2 \cdot 7^{-2} = \frac{5^2}{7^2} = \frac{25}{49}$$

Jawaban: C

$$3. \frac{125^{\frac{2}{3}} - \left(\frac{1}{3}\right)^{-2}}{2^3} = \frac{(5^3)^{\frac{2}{3}} - (3^{-1})^{-2}}{8} = \frac{5^2 - 3^2}{8} = \frac{25 - 9}{8} = 2$$

Jawaban: B

$$4. (1 + 3\sqrt{2}) - (4 - \sqrt{50}) = 1 + 3\sqrt{2} - 4 + \sqrt{25 \times 2}$$

$$= 1 - 4 + 3\sqrt{2} + 5\sqrt{2}$$

$$= -3 + 8\sqrt{2} = 8\sqrt{2} - 3$$

Jawaban: C

$$5. \frac{\sqrt{5} + 2\sqrt{3}}{\sqrt{5} - 3\sqrt{3}} \times \frac{\sqrt{5} + 3\sqrt{3}}{\sqrt{5} + 3\sqrt{3}} = \frac{5 + 3\sqrt{15} + 2\sqrt{15} + 18}{5 - 3^2 \times 3}$$

$$= \frac{23 + 5\sqrt{15}}{5 - 27} = \frac{23 + 5\sqrt{15}}{-22}$$

Jawaban: E

$$6. \frac{(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})}{4\sqrt{3} + 3\sqrt{5}} = \frac{3 - 5}{4\sqrt{3} + 3\sqrt{5}}$$

$$= \frac{-2}{4\sqrt{3} + 3\sqrt{5}} \cdot \frac{4\sqrt{3} - 3\sqrt{5}}{4\sqrt{3} - 3\sqrt{5}}$$

$$= \frac{-2(4\sqrt{3} - 3\sqrt{5})}{48 - 45}$$

$$= -\frac{1}{3}(8\sqrt{3} - 6\sqrt{5})$$

Jawaban: D

$$7. {}^2\log 3 = a \text{ dan } {}^3\log 5 = b$$

$${}^{15}\log 20 = \frac{{}^3\log 20}{{}^3\log 15} = \frac{{}^3\log(4 \times 5)}{{}^3\log(3 \times 5)} = \frac{{}^3\log 4 + {}^3\log 5}{{}^3\log 3 + {}^3\log 5}$$

$$= \frac{2 \cdot {}^3\log 2 + b}{1 + b} = \frac{2 \cdot \frac{1}{2}\log 3 + b}{1 + b}$$

$$= \frac{\frac{2}{a} + b}{1 + b} = \frac{\frac{2 + ab}{a}}{1 + b} = \frac{2 + ab}{a(1 + b)}$$

Jawaban: B

$$\begin{aligned}
8. \quad & \frac{{}^3\log 5 \cdot {}^{\sqrt{5}}\log 9 + {}^8\log 2}{{}^2\log 12 - {}^2\log 3} = \frac{{}^3\log 5 \cdot {}^{5^{\frac{1}{2}}}\log 9 + {}^{2^3}\log 2}{{}^2\log\left(\frac{12}{3}\right)} \\
& = \frac{{}^3\log 5 \cdot {}^{\frac{1}{2}}\log 9 + {}^{\frac{1}{3}}\log 2}{{}^2\log 4} = \frac{{}^3\log 5 \cdot 2 \cdot {}^5\log 9 + {}^{\frac{1}{3}} \cdot 1}{{}^2\log 2^2} \\
& = \frac{2^3 \log 9 + \frac{1}{3}}{2} = \frac{2 \cdot 2 + \frac{1}{3}}{2} = \frac{4 + \frac{1}{3}}{2} = \frac{\frac{12+1}{3}}{2} = \frac{13}{6}
\end{aligned}$$

Jawaban: D

$$\begin{aligned}
9. \quad & \left[\frac{{}^3\log 2 \cdot {}^4\log 27 + {}^3\log 81}{{}^2\log 8 - {}^2\log 4} \right]^2 = \left[\frac{\frac{3}{2} \cdot {}^3\log 2 \cdot {}^2\log 3 + 4}{3-2} \right]^2 \\
& = \left[\frac{3}{2} + 4 \right]^2 = \left(\frac{11}{2} \right)^2 = \frac{121}{4}
\end{aligned}$$

Jawaban: A

Bab 3 Persamaan dan Fungsi Kuadrat

1. $x^2 - 8x + c = 0$ akar-akarnya x_1 dan x_2 , dengan $x_2 = 3x_1$. Dari persamaan kuadrat ini diperoleh:

$$x_1 + x_2 = -\frac{-8}{1} = 8 \text{ dan } x_1 \cdot x_2 = \frac{c}{1} = c$$

Substitusikan $x_2 = 3x_1$ ke $x_1 + x_2 = 8$, diperoleh:

$$x_1 + 3x_1 = 8$$

$$4x_1 = 8$$

$$x_1 = 2$$

Substitusikan $x_1 = 2$ ke $x_2 = 3x_1$, diperoleh:

$$x_2 = 3(2) = 6$$

Jadi, $c = x_1 \cdot x_2 = 2 \cdot 6 = 12$.

Jawaban: D

2. $x^2 + (2a - 3)x + 18 = 0$ akar-akarnya p dan q , dengan $p = 2q$. Dari persamaan kuadrat ini diperoleh:

$$p + q = -\frac{(2a-3)}{1} = 3 - 2a \text{ dan } p \cdot q = \frac{18}{1} = 18$$

Substitusikan $p = 2q$ ke $p \cdot q = 18$, diperoleh

$$2q \cdot q = 18$$

$$2q^2 = 18$$

$$q^2 = 9 \Rightarrow \pm q = \sqrt{9} = \pm 3$$

karena $p > 0, q > 0$ yang dipilih $q = 3$.

Substitusikan $q = 3$ ke $p = 2q$, diperoleh:

$$p = 2(3) = 6$$

Substitusikan $p = 6$ dan $q = 3$ ke $p + q = 3 - 2a$, diperoleh:

$$6 + 3 = 3 - 2a$$

$$9 = 3 - 2a$$

$$9 - 3 = -2a$$

$$6 = -2a$$

$$2a = -6$$

$$a = -3$$

Jadi, $a - 1 = -3 - 1 = -4$.

Jawaban: B

3. Akar-akar dari persamaan kuadrat $x^2 + 4x - 3 = 0$ adalah x_1 dan x_2 , dengan:

$$x_1 + x_2 = -\frac{4}{1} = -4$$

$$x_1 \cdot x_2 = \frac{-3}{1} = -3$$

Persamaan kuadrat baru yang akar-akarnya $y_1 = 4x_1 - 1$ dan $y_2 = 4x_2 - 1$, maka:

$$y_1 + y_2 = 4x_1 - 1 + 4x_2 - 1 = 4(x_1 + x_2) - 2$$

$$= 4(-4) - 2 = -16 - 2 = -18$$

$$y_1 \cdot y_2 = (4x_1 - 1)(4x_2 - 1)$$

$$= 16x_1x_2 - 4x_1 - 4x_2 + 1$$

$$= 16x_1x_2 - 4(x_1 + x_2) + 1$$

$$= 16(-3) - 4(-4) + 1$$

$$= -48 + 16 + 1 = -31$$

Sehingga,

$$x^2 - (y_1 + y_2)x + (y_1y_2) = 0$$

$$x^2 - (-18)x + (-31) = 0$$

$$x^2 + 18x - 31 = 0$$

Jawaban: C

4. Persamaan kuadrat $x^2 + (a - 2)x + a - 2 = 0$ agar akar-akarnya nyata, maka

$$D \geq 0 \Rightarrow b^2 - 4ac \geq 0$$

$$(a - 2)^2 - 4 \cdot 1 \cdot (a - 2) \geq 0$$

$$(a - 2)(a - 2 - 4) \geq 0$$

$$(a - 2)(a - 6) \geq 0$$

Pembuat nolnya $a = 2$ dan $a = 6$, sehingga nilai yang memenuhi $a \leq 2$ atau $a \geq 6$.

Jawaban: D

5. Grafik $y = f(x) = x^2 + bx + 4$ menyinggung garis

$$y = 3x + 4, \text{ maka}$$

$$y = y \Rightarrow x^2 + bx + 4 = 3x + 4$$

$$x^2 + bx - 3x + 4 - 4 = 0$$

$$x^2 + (b - 3)x = 0$$

Syarat bersinggungan adalah $D = 0$, sehingga

$$b^2 - 4ac = 0$$

$$(b - 3)^2 - 4 \cdot 1 \cdot 0 = 0$$

$$(b - 3)^2 = 0$$

$$b = 3$$

Jawaban: D

6. Dari fungsi kuadrat $f(x) = (p - 2)x^2 + 2px + p + 3$ diperoleh:

$$a = p - 2, b = 2p, \text{ dan } c = p + 3$$

Syarat definit positif:

a. $D < 0$

$$\begin{aligned} b^2 - 4ac &< 0 \\ (2p)^2 - 4(p-2)(p+3) &< 0 \\ 4p^2 - 4(p^2 + p - 6) &< 0 \\ 4p^2 - 4p^2 - 4p + 24 &< 0 \\ -4p &< -24 \\ p &> 6 \end{aligned}$$

b. $a < 0$

$$\begin{aligned} p - 2 &< 0 \\ p &< 2 \\ a \cap b &= p > 6 \end{aligned}$$

Jadi, supaya definit positif, maka $p > 6$.

Jawaban: D

7. Dari fungsi kuadrat $f(x) = (m+1)x^2 - 2mx + (m-3)$ diperoleh:

$a = (m+1)$, $b = -2m$, dan $c = m-3$

Syarat definit negatif:

a. $D < 0$

$$\begin{aligned} b^2 - 4ac &< 0 \\ (-2m)^2 - 4(m+1)(m-3) &< 0 \\ 4m^2 - 4(m^2 - 2m - 3) &< 0 \\ 4m^2 - 4m^2 + 8m + 12 &< 0 \\ 8m &< -12 \\ m &< -\frac{3}{2} \end{aligned}$$

b. $a < 0$

$$\begin{aligned} m + 1 &< 0 \\ m &< -1 \end{aligned}$$

$$a \cap b = m < -\frac{3}{2}$$

Jadi, supaya definit negatif, maka $m < -\frac{3}{2}$.

Jawaban: A

Bab 4 Sistem Persamaan Linear

1. Aprilia : $4A + 2B = 4.000$... (1)

Julia : $10A + 4B = 9.500$... (2)

Januar : $A + B = ?$

$$\begin{array}{r|l} 4A + 2B = 4.000 & \times 2 \\ 10A + 4B = 9.500 & \times 1 \\ \hline -2A = -1.500 & - \\ A = 750 & \end{array}$$

Substitusikan $A = 750$ ke $4A + 2B = 4.000$, diperoleh:

$$4(750) + 2B = 4.000$$

$$3.000 + 2B = 4.000$$

$$2B = 4.000 - 3.000 = 1.000$$

$$B = 500$$

$$A + B = 750 + 500 = 1.250$$

Dengan demikian, harga barang yang dibeli Januar adalah Rp1.250,00.

Jawaban: D

2. Misalkan buku = x , pulpen = y , dan pensil = z

Adil : $4x + 2y + 3z = 26.000$... (1)

Bima : $3x + 3y + z = 21.500$... (2)

Citra : $3x + z = 12.500$... (3)

Persamaan (2) dan (3)

$$\begin{array}{r} 3x + 3y + z = 21.500 \\ 3x + z = 12.500 \quad - \\ \hline 3y = 9.000 \\ y = 3.000 \end{array}$$

Substitusikan nilai $y = 3.000$ ke persamaan (1) dan (2), diperoleh:

$$\begin{aligned} (1) \quad 4x + 2y + 3z &= 26.000 \\ \Rightarrow 4x + 2(3.000) + 3z &= 26.000 \\ \Rightarrow 4x + 6.000 + 3z &= 26.000 \\ \Rightarrow 4x + 3z &= 26.000 - 6.000 \\ \Rightarrow 4x + 3z &= 20.000 \quad \dots (4) \end{aligned}$$

$$\begin{aligned} (2) \quad 3x + 3y + z &= 21.500 \\ \Rightarrow 3x + 3(3.000) + z &= 21.500 \\ \Rightarrow 3x + 9.000 + z &= 21.500 \\ \Rightarrow 3x + z &= 21.500 - 9.000 \\ \Rightarrow 3x + z &= 12.500 \quad \dots (5) \end{aligned}$$

Eliminasi (4) dan (5), diperoleh:

$$\begin{array}{r|l} 4x + 3z = 20.000 & \times 1 \\ 3x + z = 12.500 & \times 3 \\ \hline -5x = -17.500 & - \\ \Rightarrow x = \frac{-17.500}{-5} & \\ \Rightarrow x = 3.500 & \end{array}$$

$x = 3.500$ disubstitusikan ke persamaan $3x + z = 12.500$, diperoleh

$$\begin{aligned} 3(3.500) + z &= 12.500 \Leftrightarrow 10.500 + z = 12.500 \\ \Leftrightarrow z &= 12.500 - 10.500 \\ \Leftrightarrow z &= 2.000 \end{aligned}$$

Dina membeli 2 pulpen dan 2 pensil, sehingga

$$\begin{aligned} 2y + 2z &= 2(3.500) + 2(2.000) \\ &= 7.000 + 4.000 = 11.000 \end{aligned}$$

Jadi, Dina harus membayar Rp11.000,00

Jawaban: D

3. Misalkan harga 1 kg bakso sapi = x dan harga 1 kg bakso ikan = y .

Ibu Aya : $4x + 5y = 530.000$... (1)

Ibu Beta : $5x + 4y = 550.000$... (2)

Ibu Cita : $2x + 3y = ?$

Berdasarkan persamaan (1) dan (2), diperoleh:

$$\begin{array}{rcl} 4x + 5y = 530.000 & \times 4 & 16x + 20y = 2.120.000 \\ 5x + 4y = 550.000 & \times 5 & 25x + 20y = 2.750.000 \\ \hline -9x = -630.000 & & \\ x = 70.000 & & \end{array}$$

Substitusikan $x = 70.000$ ke persamaan (1), diperoleh:

$$\begin{aligned} 4x + 5y &= 530.000 \\ 4(70.000) + 5y &= 530.000 \\ 280.000 + 5y &= 530.000 \\ 5y &= 250.000 \\ y &= 50.000 \end{aligned}$$

Harga 3 kg bakso ikan dan 2 kg bakso sapi:

$$\begin{aligned} 2x + 3y &= 2(70.000) + 3(50.000) \\ &= 140.000 + 150.000 \\ &= 290.000 \end{aligned}$$

Jadi, uang yang harus dibayarkan oleh Ibu Cita adalah Rp290.000,00.

Jawaban: B

Persamaan garis singgung lingkaran

$x^2 + y^2 - 6x + 4y - 12 = 0$ di titik (7, 1) adalah

$$\begin{aligned} 7 \cdot x + 1 \cdot y - \frac{1}{2} \cdot 6(x+7) + \frac{1}{2} \cdot 4(y+1) - 12 &= 0 \\ 7x + y - 3(x+7) + 2(y+1) - 12 &= 0 \\ 7x + y - 3x - 21 + 2y + 2 - 12 &= 0 \\ 4x + 3y - 31 &= 0 \end{aligned}$$

Jawaban: D

4. $-2x + y + 5 = 0 \Leftrightarrow y = 2x - 5, m_1 = 2$

Sejajar, maka $m_2 = m_1 = 2$

$$x^2 + y^2 - 6x + 10y - 46 = 0$$

$$x^2 - 6x + 9 + y^2 + 10y + 25 = 46 + 9 + 25$$

$$(x-3)^2 + (y+5)^2 = 80$$

Garis singgung lingkaran:

$$y - b = m(x - a) \pm r\sqrt{1 + m^2}$$

$$y + 5 = 2(x - 3) \pm \sqrt{80}\sqrt{1 + 2^2}$$

$$y + 5 = 2x - 6 \pm \sqrt{80}(5)$$

$$y = 2x - 11 \pm 20$$

Jawaban: E

Bab 5 Persamaan Lingkaran

1. Pusat (3, -2) dan menyinggung sumbu-Y, maka $r = 3$
Persamaan lingkaran:

$$(x - a)^2 + (y - b)^2 = r^2$$

$$(x - 3)^2 + (y + 2)^2 = 3^2$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 = 9$$

$$x^2 + y^2 - 6x + 4y + 4 = 0$$

Jawaban: D

2. $3x - y = 0 \Leftrightarrow y = 3x, m_1 = 3$

Tegak lurus $\Leftrightarrow m_1 \cdot m_2 = -1$

$$3 \cdot m_2 = -1 \Leftrightarrow m_2 = -\frac{1}{3}$$

$$x^2 + y^2 - 2x - 6y + 1 = 0$$

$$(x - 1)^2 - 1 + (y - 3)^2 - 9 + 1 = 0$$

$$(x - 1)^2 + (y - 3)^2 = 9$$

Garis singgung lingkaran:

$$y - b = m(x - a) \pm r\sqrt{1 + m^2}$$

$$y - 3 = -\frac{1}{3}(x - 1) \pm 3\sqrt{1 + \frac{1}{9}}$$

$$y - 3 = -\frac{1}{3}(x - 1) \pm \sqrt{10}$$

Jawaban: C

3. Persamaan garis singgung lingkaran $x^2 + y^2 + Ax + By + C = 0$ di titik (x_1, y_1) adalah

$$x_1 \cdot x + y_1 \cdot y + \frac{1}{2}A(x + x_1) + \frac{1}{2}B(y + y_1) + C = 0$$

Bab 6 Suku Banyak

1. $P(x) = 2x^4 + ax^3 - 3x^2 + 5x + b$

$$P(1) = 2(1)^4 + a(1)^3 - 3(1)^2 + 5(1) + b = 11$$

$$2 + a - 3 + 5 + b = 11$$

$$a + b = 7 \quad \dots (1)$$

$$P(-1) = 2(-1)^4 + a(-1)^3 - 3(-1)^2 + 5(-1) + b = -1$$

$$2 - a - 3 - 5 + b = -1$$

$$-a + b = 5 \quad \dots (2)$$

$$(1) \quad a + b = 7$$

$$(2) \quad \frac{-a + b = 5}{2b = 12} \Rightarrow b = 6$$

$$2b = 12 \Rightarrow b = 6$$

Substitusikan $b = 6$ ke $a + b = 7$, diperoleh:

$$a + 6 = 7$$

$$a = 1$$

$$\text{Jadi, nilai } 2a + b = 2(1) + 6 = 2 + 6 = 8$$

Jawaban: C

2. $f(x) = h(x)(x - 2) + 11$, maka $f(2) = 11$

$$f(x) = h(x)(x + 1) - 4, \text{ maka } f(-1) = -4$$

$$f(x) = h(x)(x^2 - x - 2) + Ax + B$$

$$f(x) = h(x)(x - 2)(x + 1) + Ax + B$$

$$f(2) = 0 + 2A + B = 11$$

$$f(-1) = 0 - A + B = -4$$

$$3A = 15 \Rightarrow A = 5, \text{ sehingga } B = 1$$

$$\text{Jadi, sisanya adalah } Ax + B = 5x + 1$$

Jawaban: D

$$\begin{aligned}
 3. \quad f(x) &= h(x)(x^2 - x - 2) + ax + b \\
 &= h(x)(x - 2)(x + 1) + ax + b \\
 f(2) &= 2a + b = 11 \quad \dots(1) \\
 f(-1) &= -a + b = -4 \quad \dots(2)
 \end{aligned}$$

Berdasarkan persamaan (1) dan (2) diperoleh:

$$\begin{array}{r}
 2a + b = 11 \\
 -a + b = -4 \\
 \hline
 3a = 15 \\
 a = 5
 \end{array}$$

Substitusikan $a = 5$ ke persamaan (1), diperoleh:

$$\begin{aligned}
 2a + b &= 11 \\
 2(5) + b &= 11 \\
 10 + b &= 11 \\
 b &= 1
 \end{aligned}$$

Sehingga, $s(x) = ax + b = 5x + 1$

Jawaban: D

4. Faktor dari $2x^3 + px^2 - 10x - 24$ adalah $x + 4$, sehingga

$$\begin{aligned}
 x = -4 \Rightarrow 2(-4)^3 + p(-4)^2 - 10(-4) - 24 &= 0 \\
 -128 + 16p + 40 - 24 &= 0 \\
 p &= 7
 \end{aligned}$$

$$\begin{array}{r|rrrr}
 x = -4 & 2 & 7 & -10 & -24 \\
 & & -8 & 4 & 24 \\
 \hline
 & 2 & -1 & -6 & 0
 \end{array}$$

Faktor yang lainnya diperoleh dari

$$2x^2 - x - 6 = (2x + 3)(x - 2)$$

Jawaban: E

$$\begin{aligned}
 5. \quad f(x) &= h(x)(x^2 - 3x + 2) + 3x + 2 \\
 &= h(x)(x - 2)(x - 1) + 3x + 2 \\
 f(2) &= 3(2) + 2 = 8 \\
 f(1) &= 3(1) + 2 = 5
 \end{aligned}$$

Kita pilih $f(1) = 5$, lalu substitusikan ke dalam pilihan jawaban. Pilihan jawaban yang memenuhi adalah jawaban C, karena $f(1) = 1^3 + 1^2 - 7(1) + 10 = 5$

Jawaban: C

Misalkan $y = 2x - 4$, sehingga $x = \frac{1}{2}y + 2$

$$\begin{aligned}
 g(y) &= 4\left(\frac{1}{2}y + 2\right)^2 - 24\left(\frac{1}{2}y + 2\right) + 32 \\
 &= 4\left(\frac{1}{4}y^2 + 2y + 4\right) - 12y - 48 + 32 \\
 &= y^2 - 4y
 \end{aligned}$$

Sehingga $g(x) = x^2 - 4x$.

Jawaban: E

$$\begin{aligned}
 3. \quad (g \circ f)(x) &= g(f(x)) = 2x^2 + 4x + 5 \\
 2f(x) + 3 &= 2x^2 + 4x + 5 \\
 2f(x) &= 2x^2 + 4x + 2 \\
 f(x) &= x^2 + 2x + 1
 \end{aligned}$$

Jawaban: A

$$4. \text{ Invers dari } f(x) = \frac{ax+b}{cx+d} \text{ adalah } f^{-1}(x) = \frac{-dx+b}{cx-a}$$

$$\text{Sehingga } f(x) = \frac{3x+1}{2x-2} \Rightarrow f^{-1}(x) = \frac{2x+1}{2x-3}, x \neq \frac{3}{2}$$

Jawaban: E

$$\begin{aligned}
 5. \quad (f \circ g)(x) &= f(g(x)) = f(2x - 1) = \frac{2x - 1 + 4}{2x - 1 - 6} = \frac{2x + 3}{2x - 7} \\
 (f \circ g)^{-1}(x) &= \frac{7x + 3}{2x - 2}, x \neq 1
 \end{aligned}$$

Jawaban: C

$$\begin{aligned}
 6. \quad (f \circ g)(x) &= f(g(x)) = f(x - 3) \\
 &= \frac{3(x - 3) + 1}{(x - 3) - 5} = \frac{3x - 9 + 1}{x - 3 - 5} \\
 &= \frac{3x - 8}{x - 8}, x \neq 8
 \end{aligned}$$

$$\text{Invers dari } f(x) = \frac{ax+b}{cx+d} \text{ adalah } f^{-1}(x) = \frac{-dx+b}{cx-a}$$

Sehingga,

$$\begin{aligned}
 (f \circ g)(x) &= \frac{3x - 8}{x - 8} \Rightarrow (f \circ g)^{-1}(x) = \frac{-(-8)x - 8}{x - 3} \\
 &= \frac{8x - 8}{x - 3}, x \neq 3
 \end{aligned}$$

Jawaban: D

Bab 7 Komposisi Fungsi dan Invers Fungsi

$$\begin{aligned}
 1. \quad (f \circ g)(x) &= f(g(x)) \\
 &= 2\left(\frac{x-1}{x+4}\right) + 5 = \frac{2x-2}{x+4} + 5 \cdot \frac{x+4}{x+4} \\
 &= \frac{2x-2}{x+4} + \frac{5x+20}{x+4} = \frac{7x+18}{x+4}, x \neq -4
 \end{aligned}$$

Jawaban: D

$$\begin{aligned}
 2. \quad (g \circ f)(x) &= 4x^2 - 24x + 32 \\
 g(f(x)) &= 4x^2 - 24x + 32 \\
 g(2x - 4) &= 4x^2 - 24x + 32
 \end{aligned}$$

Bab 8 Program Linear

1. Langkah 1: Tabel

	Harga Beli	Muatan	Keuntungan
Jeruk (x)	5.000	x	6.500 - 5.000 = 1.500
Mangga (y)	6.000	y	8.000 - 6.000 = 2.000
Jumlah	600.000	110	1.500x + 2.000y

Langkah 2: Model Matematika

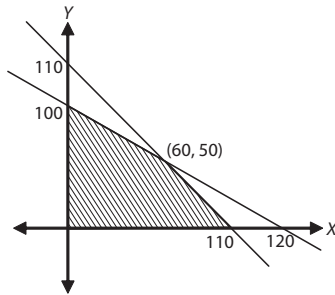
Maks: $f(x, y) = 1.500x + 2.000y$

$5.000x + 6.000y \leq 600.000 \Leftrightarrow 5x + 6y \leq 600$

$x + y \leq 110, x \geq 0, y \geq 0$

Langkah 3: Penyelesaian

Gambar dengan garis pada diagram Kartesius



$$5x + 6y = 600$$

$$5x + 5y = 550 \quad -$$

$$y = 50, \text{ maka } x = 60$$

Titik Pojok	$f(x, y) = 1.500x + 2.000y$
(0, 0)	0
(110, 0)	165.000
(60, 50)	190.000
(0, 100)	200.000

Jadi, laba maksimum yang diperoleh penjual adalah Rp200.000,00.

Jawaban: C

2. Langkah 1: Tabel

Jenis Barang	Mesin A	Mesin B	Keuntungan
Model I (x)	2 jam	1 jam	40.000
Model II (y)	1 jam	5 jam	10.000
	12 jam	15 jam	$40.000x + 10.000y$

Langkah 2: Model Matematika

Maks: $f(x, y) = 40.000x + 10.000y$

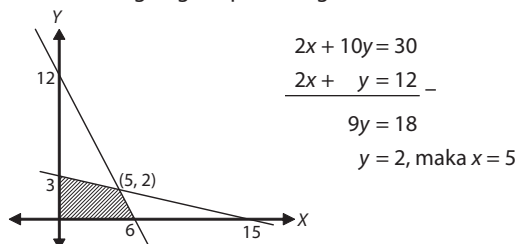
$x \geq 0, y \geq 0$

$2x + y \leq 12,$

$x + 5y \leq 15,$

Langkah 3: Penyelesaian

Gambar dengan garis pada diagram Kartesius



$$2x + 10y = 30$$

$$2x + y = 12 \quad -$$

$$9y = 18$$

$$y = 2, \text{ maka } x = 5$$

Titik Pojok	$f(x, y) = 40.000x + 10.000y$
(0, 0)	0
(6, 0)	240.000
(5, 2)	220.000
(0, 3)	30.000

Jadi, keuntungan maksimum yang diperoleh perusahaan adalah Rp240.000,00.

Jawaban: C

3. Misalkan baju model I = x dan baju model II = y.

Pakaian	Kain Polos (m)	Kain Bergaris (m)
Model I	1	3
Model II	2	1
Persediaan	20	20

Model matematika:

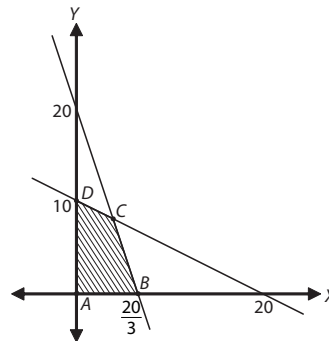
$x + 2y \leq 20$

$3x + y \leq 20$

$x \geq 0$

$y \geq 0$

Fungsi objektif: $150.000x + 100.000y$



Titik potong C

$$x + 2y = 20 \quad \times 1 \quad x + 2y = 20$$

$$3x + y = 20 \quad \times 2 \quad 6x + 2y = 40 \quad -$$

$$-5x = -20$$

$$x = 4$$

$$x = 4 \rightarrow x + 2y = 20$$

$$4 + 2y = 20$$

$$2y = 16$$

$$y = 8$$

Sehingga, C(4, 8)

Uji titik pojok:

$$B\left(\frac{20}{3}, 0\right) \rightarrow f = 150.000\left(\frac{20}{3}\right) + 100.000(0) = 1.000.000$$

$$C(4, 8) \rightarrow f = 150.000(4) + 100.000(8) = 1.400.000$$

$$D(0, 10) \rightarrow f = 150.000(0) + 100.000(10) = 1.000.000$$

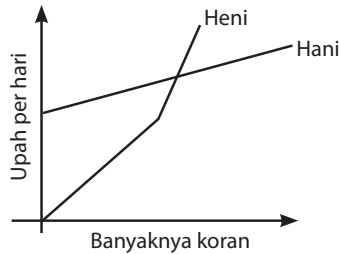
Jadi, penghasilan maksimum yang dapat diperoleh penjahit tersebut adalah Rp1.400.000,00.

Jawaban: A

4. Fungsi upah Heni $\Rightarrow f(x) = \begin{cases} 1.500x, & 0 \leq x \leq 100 \\ 2.000x + 5.000, & x > 100 \end{cases}$

Fungsi upah Hani $g(x) = 10.000 + 100x, x \geq 0$

Grafik kedua fungsi tersebut adalah



Jawaban: D

Bab 9 Matriks

1. $A + 3B = C$

$$\begin{pmatrix} 4 & -1 \\ -2 & 7 \end{pmatrix} + \begin{pmatrix} -12 & 3 \\ 6 & -21 \end{pmatrix} = \begin{pmatrix} -8 & a \\ b & -14 \end{pmatrix}$$

$$\begin{pmatrix} -8 & 2 \\ 4 & -14 \end{pmatrix} = \begin{pmatrix} -8 & a \\ b & -14 \end{pmatrix}$$

Dari kesamaan matriks diperoleh $a = 2$ dan $b = 4$.

Jawaban: A

2. $\begin{pmatrix} x-5 & 4 \\ -5 & 2 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 2 & y-1 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -16 & 5 \end{pmatrix}$
- $$\begin{pmatrix} 4x-20+8 & -x+5+4y-4 \\ -20+4 & 5+2y-2 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -16 & 5 \end{pmatrix}$$
- $$\begin{pmatrix} 4x-12 & -x+4y+1 \\ -16 & 2y+3 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -16 & 5 \end{pmatrix}$$

Dari kesamaan matriks diperoleh

$$\begin{aligned} 4x - 12 &= 0 & 2y + 3 &= 5 \\ 4x &= 12 & 2y &= 2 \\ x &= 3 & y &= 1 \end{aligned}$$

Jadi, perbandingan x dan y adalah $3 : 1$.

Jawaban: A

3. $A = 2B^T \Leftrightarrow \begin{pmatrix} a & 4 \\ 2b & 3c \end{pmatrix} = 2 \begin{pmatrix} 2c-3b & a \\ 2a+1 & b+7 \end{pmatrix}$

Diperoleh kesamaan:

$$\begin{aligned} 4 &= 2a & \dots (1) \\ 2b &= 2(2a+1) & \dots (2) \\ 3c &= 2(b+7) & \dots (3) \end{aligned}$$

Dari (1) diperoleh $a = 2$, substitusikan ke (2) diperoleh $2b = 2(4+1) \Leftrightarrow b = 5$, substitusikan ke (3) diperoleh $3c = 2(5+7) \Leftrightarrow c = 8$

Nilai $a + b + c = 2 + 5 + 8 = 15$.

Jawaban: D

4. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$
- $$X = \frac{1}{4-6} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$$
- $$X = -\frac{1}{2} \begin{pmatrix} 16-4 & 12-2 \\ -12+2 & -9+1 \end{pmatrix}$$
- $$X = -\frac{1}{2} \begin{pmatrix} 12 & 10 \\ -10 & -8 \end{pmatrix} = \begin{pmatrix} -6 & -5 \\ 5 & 4 \end{pmatrix}$$
- $$\det(X) = \begin{vmatrix} -6 & -5 \\ 5 & 4 \end{vmatrix} = (-6)(4) - (-5)(5)$$
- $$= -24 + 25 = 1$$

Jawaban: C

5. $A - B = \begin{pmatrix} 3 & -2 \\ -2 & -2 \end{pmatrix} - \begin{pmatrix} 2 & -2 \\ -2 & -3 \end{pmatrix}$
- $$= \begin{pmatrix} 3-2 & -2-(-2) \\ -2-(-2) & -2-(-3) \end{pmatrix}$$
- $$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
- $$(A-B)^{-1} = \frac{1}{1-0} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Jawaban: C

Bab 10 Vektor

1. $\vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$

$$\begin{pmatrix} p \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix} = 4p - 6 - 6 = 0$$

$$4p = 12$$

$$p = 3$$

Jadi,

$$(\vec{a} - 2\vec{b}) \cdot (3\vec{c}) = \left[\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} - 2 \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix} \right] \cdot 3 \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

$$= \left[\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 8 \\ -6 \\ 12 \end{pmatrix} \right] \cdot \begin{pmatrix} 6 \\ -3 \\ 9 \end{pmatrix} = \begin{pmatrix} -5 \\ 8 \\ -13 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -3 \\ 9 \end{pmatrix}$$

$$= (-5)(6) + (8)(-3) + (-13)(9)$$

$$= -30 - 24 - 117 = -171$$

Jawaban: E

2. $\vec{AB} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ dan $\vec{AC} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

Misalkan α adalah sudut antara $\vec{AB}$ dan $\vec{AC}$, maka

$$\cos \alpha = \frac{\overline{AB} \cdot \overline{AC}}{|\overline{AB}| |\overline{AC}|}$$

$$= \frac{\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}}{\sqrt{(-1)^2 + 1^2 + (-1)^2} \cdot \sqrt{0^2 + 1^2 + (-1)^2}} \\ = \frac{0+1+1}{\sqrt{3} \cdot \sqrt{2}} = \frac{2}{\sqrt{6}} = \frac{2}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{2}{6} \sqrt{6} = \frac{1}{3} \sqrt{6}$$

Jawaban: A

$$3. \quad \vec{a} = 4\vec{i} - 2\vec{j} + 2\vec{k} = \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} \text{ dan } \vec{b} = 2\vec{i} - 6\vec{j} + 4\vec{k} = \begin{pmatrix} 2 \\ -6 \\ 4 \end{pmatrix}$$

Proyeksi vektor $\vec{a}$ pada $\vec{b} = \vec{a}_b$

$$\vec{a}_b = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right) \cdot \vec{b} = \frac{\begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -6 \\ 4 \end{pmatrix}}{(\sqrt{2^2 + (-6)^2 + 4^2})^2} \cdot \begin{pmatrix} 2 \\ -6 \\ 4 \end{pmatrix} \\ = \frac{8+12+8}{(\sqrt{4+36+16})^2} \cdot \begin{pmatrix} 2 \\ -6 \\ 4 \end{pmatrix} = \frac{28}{56} \cdot \begin{pmatrix} 2 \\ -6 \\ 4 \end{pmatrix} = \frac{1}{2} \cdot \begin{pmatrix} 2 \\ -6 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \\ = \vec{i} - 3\vec{j} + 2\vec{k}$$

Jawaban: B

$$4. \quad \vec{a} = -\vec{i}\sqrt{3} + p\vec{j} + \vec{k} = \begin{pmatrix} -\sqrt{3} \\ p \\ 1 \end{pmatrix} \\ \vec{b} = -\vec{i}\sqrt{3} + 2\vec{j} + p\vec{k} = \begin{pmatrix} -\sqrt{3} \\ 2 \\ p \end{pmatrix}$$

Panjang proyeksi orthogonal vektor $\vec{a}$ pada $\vec{b} = |\vec{a}_b|$,

$$\text{sehingga } |\vec{a}_b| = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$\frac{3}{2} = \frac{\begin{pmatrix} -\sqrt{3} \\ p \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -\sqrt{3} \\ 2 \\ p \end{pmatrix}}{\sqrt{(-\sqrt{3})^2 + 2^2 + p^2}} \\ \frac{3}{2} = \frac{3+2p+p}{\sqrt{p^2+7}}$$

$$3\sqrt{p^2+7} = 6+6p$$

$$\sqrt{p^2+7} = 2+2p$$

$$7 + p^2 = 4 + 8p + 4p^2$$

$$3p^2 + 8p - 3 = 0$$

$$(3p-1)(p+3) = 0$$

$$p_1 = \frac{1}{3} \text{ atau } p_2 = -3 \text{ (tidak memenuhi)}$$

Jawaban: C

Bab 11 Transformasi

1. Matriks yang bersesuaian dengan refleksi terhadap garis

$$y = -x, \text{ yaitu } T_1 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

Matriks yang bersesuaian dengan refleksi terhadap garis

$$y = x, \text{ yaitu } T_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Matriks refleksi terhadap garis $y = -x$ dilanjut $y = x$, bersesuaian dengan

$$T_2 \circ T_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\text{Sehingga } \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \\ \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$$

$$\text{Dengan demikian } x' = -x \Rightarrow x = -x' \\ y' = -y \Rightarrow y = -y'$$

$$y = 2x - 3 \xrightarrow{T_2 \circ T_1} (-y') = 2(-x') - 3 \\ y - 2x - 3 = 0$$

Jawaban: B

$$2. \quad \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \\ = \begin{pmatrix} 3-2 & -3+4 \\ 2-1 & -2+2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-1} \begin{pmatrix} 0 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \\ = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} y' \\ x' - y' \end{pmatrix}$$

Bayangan $3x + 5y - 7 = 0$ adalah

$$3y' + 5(x' - y') - 7 = 0$$

$$3y + 5x - 5y - 7 = 0$$

$$5x - 2y - 7 = 0$$

Jawaban: D

$$\begin{aligned}
3. \quad \begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \\
\begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} 3 & 5 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \\
\begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 & 5 \\ -1 & -3 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \\
\begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2x' + 5y' \\ -x' - 3y' \end{pmatrix}
\end{aligned}$$

Diperoleh:

$$x = 2x' + 5y'$$

$$y = -x' - 3y'$$

Sehingga, persamaan bayangannya:

$$(2x' + 5y') - 2(-x' - 3y') = 5$$

$$2x + 5y + 2x + 6y = 5$$

$$4x + 11y = 5$$

Jawaban: C

Bab 12 Eksponen dan Logaritma

$$1. \quad \frac{1}{2} \log(x^2 - 8) < 0$$

Syarat (1)

$$\frac{1}{2} \log(x^2 - 8) < \frac{1}{2} \log\left(\frac{1}{2}\right)^0$$

$$\begin{aligned}
x^2 - 8 &> 1 & \left(\begin{array}{l} \text{tanda berubah, bilangan} \\ \text{basisnya di antara 0 dan 1} \end{array} \right) \\
x^2 - 8 - 1 &> 0 \\
x^2 - 9 &> 0
\end{aligned}$$

$$(x - 3)(x + 3) > 0$$

$$x < -3 \text{ atau } x > 3 \quad \dots(1)$$

Syarat (2)

$$x^2 - 8 > 0$$

$$(x - 2\sqrt{2})(x + 2\sqrt{2}) > 0$$

$$x < -2\sqrt{2} \text{ atau } x > 2\sqrt{2} \quad \dots(2)$$

Jadi, $(1) \cap (2)$ diperoleh $x < -3$ atau $x > 3$.

Jawaban: C

$$2. \quad {}^{25}\log(x-3) + {}^{25}\log(x+1) \leq \frac{1}{2}$$

Syarat (1)

$${}^{25}\log(x-3) + {}^{25}\log(x+1) \leq \frac{1}{2}$$

$${}^{25}\log(x-3)(x+1) \leq {}^{25}\log 5$$

$${}^{25}\log(x^2 - 2x - 3) \leq {}^{25}\log 5$$

$$x^2 - 2x - 3 \leq 5$$

$$x^2 - 2x - 8 \leq 0$$

$$(x - 4)(x + 2) \leq 0$$

$$-2 \leq x \leq 4 \quad \dots(1)$$

Syarat (2)

$$x - 3 > 0$$

$$x > 3 \quad \dots(2)$$

Syarat (3)

$$x + 1 > 0$$

$$x > -1 \quad \dots(3)$$

$(1) \cap (2) \cap (3)$ diperoleh $3 < x \leq 4$

Jawaban: D

$$\begin{aligned}
3. \quad 9^{2x-4} &\geq \left(\frac{1}{27}\right)^{x^2-4} \\
(3^2)^{2x-4} &\geq (3^{-3})^{x^2-4} \\
3^{4x-8} &\geq 3^{-3x^2+12} \\
4x-8 &\geq -3x^2+12 \\
4x-8+3x^2-12 &\geq 0 \\
3x^2+4x-20 &\geq 0
\end{aligned}$$

Pembuat nol

$$(3x + 10)(x - 2) = 0$$

$$x_1 = -\frac{10}{3}, \quad x_2 = 2$$

Himpunan penyelesaian = $\left\{x \mid x \leq -\frac{10}{3} \text{ atau } x \geq 2\right\}$

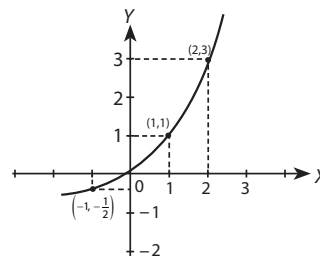
Jawaban: C

$$\begin{aligned}
4. \quad \left(\frac{1}{3}\right)^{2x+1} &> \sqrt{\frac{27}{3^{x-1}}} \\
(3^{-1})^{2x+1} &> \sqrt{\frac{3^3}{3^{x-1}}} \\
3^{-2x-1} &> \sqrt{3^{3-x+1}} \\
3^{-2x-1} &> 3^{\frac{4-x}{2}} \\
-2x-1 &> \frac{4-x}{2} \\
-4x-2 &> 4-x \\
-3x &> 6 \\
x &< -2
\end{aligned}$$

Penyelesaian $x < -2$.

Jawaban: D

5. Perhatikan gambar berikut.



Kurva melalui titik (2, 3). Jawaban yang memenuhi titik (2, 3) adalah hanya pilihan B. Jadi, jawaban yang benar adalah B.

Jawaban: B

Bab 13 Barisan dan Deret

1. $U_7 = U_4 + 3b = 29$

$$17 + 3b = 29$$

$$3b = 12, \text{ maka } b = 4$$

$$\text{Jadi, } U_{25} = U_7 + 18b = 29 + 18(4) = 29 + 72 = 101$$

Jawaban: B

2. $S_n = n^2 + \frac{5}{2}n$ $S' = 2n + \frac{5}{2}$
 $r = \text{koefisien } n = 2$

Jawaban: C

3. $S_{10} = \frac{10}{2} \{2 \cdot 120 + (10-1)10\}$
 $= 5 \{240 + (9)10\}$
 $= 5 \{240 + 90\} = 5 \{330\} = 1.650$

Jawaban: D

4. Banyak kursi pada baris terdepan, $U_1 = 20$ dengan $b = 4$

$$S_{15} = \frac{15}{2} \{2 \times 20 + (15-1)4\}$$

$$= \frac{15}{2} \{40 + (14)4\}$$

$$= \frac{15}{2} \{40 + 56\}$$

$$= \frac{15}{2} \{96\} = 15 \times 48 = 720$$

Jadi, kapasitas gedung pertunjukan adalah 720 tempat duduk.

Jawaban: C

5. Barisan geometri, dengan $r > 1$

$$U_5 = U_3 \cdot r^2 = 32$$

$$r^2 = \frac{32}{8} \Rightarrow r^2 = 4$$

$$U_7 = U_5 \cdot r^2 = 32 \cdot 4 = 128$$

Jawaban: C

6. $S_n = 2^{n+2} - 4$

$$U_1 = S_1 = 2^{1+2} - 4 = 8 - 4 = 4$$

$$U_1 + U_2 = S_2 = 2^4 - 4$$

$$4 + U_2 = 12, \text{ maka } U_2 = 8$$

$$\text{Jadi, } r = \frac{U_2}{U_1} = \frac{8}{4} = 2$$

Jawaban: C

7. Potongan tali terpendek, $U_1 = a = 6$

$$\text{Potongan tali terpanjang, } U_7 = ar^6 = 384$$

$$ar^6 = 384$$

$$6r^6 = 384$$

$$r^6 = 64$$

$$r = 2$$

$$S_7 = \frac{6(2^7 - 1)}{2 - 1} = \frac{6(128 - 1)}{1}$$

$$= 6(127) = 762$$

Jadi, panjang keseluruhan tali adalah 762 cm.

Jawaban: D

Bab 14 Dimensi Tiga

1. Kubus dengan rusuk 10 cm, jarak titik F ke garis AC .

$$FA = FC = AC = 10\sqrt{2}$$

Karena $FA = FC$, maka

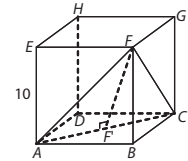
$$AF' = \frac{1}{2} AC = \frac{1}{2} \times 10\sqrt{2} = 5\sqrt{2}$$

Jarak titik F ke garis AC adalah

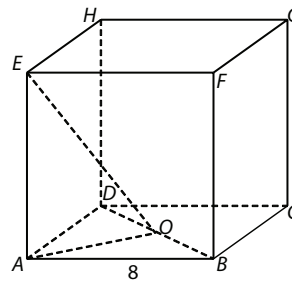
$$FF' = \sqrt{AF^2 - (AF')^2}$$

$$= \sqrt{(10\sqrt{2})^2 - (5\sqrt{2})^2} = \sqrt{200 - 50} = \sqrt{150} = 5\sqrt{6}$$

Jawaban: C



2. Kubus dengan rusuk 8 cm.



$$d(E, DB) = d(E, O) = EO$$

$$DB = AC = 8\sqrt{2}$$

$$AO = \frac{1}{2} AC = \frac{1}{2} (8\sqrt{2}) = 4\sqrt{2}$$

$$EO = \sqrt{AE^2 + AO^2} = \sqrt{(8)^2 + (4\sqrt{2})^2}$$

$$= \sqrt{64 + 32} = \sqrt{96} = 4\sqrt{6}$$

Jawaban: D

3. Pada kubus $ABCD.EFGH$, jarak titik C dengan bidang BDG adalah $\frac{1}{3} \times \text{diagonal ruang}$.

Diagonal ruang kubus dengan rusuk a cm = $a\sqrt{3}$ cm, sehingga

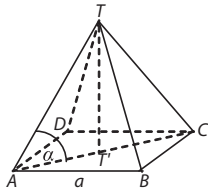
$$\text{Diagonal ruang} = 6\sqrt{3} \text{ cm}$$

Jadi, jarak titik C dengan bidang BDG adalah

$$\frac{1}{3} \times 6\sqrt{3} = 2\sqrt{3} \text{ cm.}$$

Jawaban: B

4. Sudut garis TA dan bidang $ABCD$.



Proyeksi TA pada bidang $ABCD$ adalah $T'A$

Sehingga $\angle(TA, ABCD) = \angle(TA, AT') = \alpha$

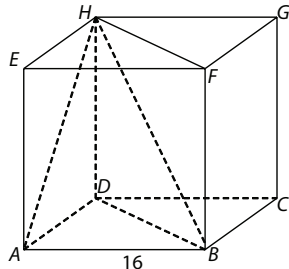
$$AT' = \frac{1}{2} AC = \frac{1}{2} a\sqrt{2}$$

Perhatikan $\triangle AT'T$

$$\cos \alpha = \frac{AT'}{AT} = \frac{\frac{1}{2} a\sqrt{2}}{\frac{1}{2} a\sqrt{2}} = \frac{1}{2} \sqrt{2} = \frac{1}{2} \sqrt{2} \Rightarrow \alpha = 45^\circ$$

Jawaban: C

5. Perhatikan kubus $ABCD.EFGH$ di bawah ini.



$$\angle(AH, BDHF) = \angle(AH, HB) = \angle H$$

$$AH = 16\sqrt{2} \text{ dan } HB = 16\sqrt{3}$$

Perhatikan segitiga AHB . Berdasarkan aturan kosinus diperoleh:

$$AB^2 = AH^2 + HB^2 - 2 \cdot AH \cdot HB \cdot \cos H$$

$$16^2 = (16\sqrt{2})^2 + (16\sqrt{3})^2 - 2(16\sqrt{2})(16\sqrt{3}) \cos H$$

$$256 = 512 + 768 - 512\sqrt{6} \cos H$$

$$\cos H = \frac{1.280 - 256}{512\sqrt{6}} = \frac{1.024}{512\sqrt{6}} = \frac{2}{\sqrt{6}} = \frac{1}{3}\sqrt{6}$$

$$\sin^2 H + \cos^2 H = 1$$

$$\sin^2 H + \left(\frac{1}{3}\sqrt{6}\right)^2 = 1$$

$$\sin^2 H + \frac{2}{3} = 1$$

$$\sin^2 H = \frac{1}{3}$$

$$\sin H = \sqrt{\frac{1}{3}} = \frac{1}{3}\sqrt{3}$$

Jawaban: B

6. Garis potong bidang ACF dan bidang $ABCD$ adalah AC . Garis pada bidang ACF dan $ABCD$ yang tegak lurus AC adalah FT dan BT .

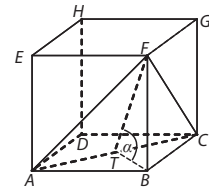
$$\angle(ACF, ABCD) = \angle(FT, BT) = \alpha$$

Perhatikan segitiga siku-siku BTF .

$$BT = \frac{1}{2} BD = \frac{1}{2} a\sqrt{2}$$

$$\begin{aligned} FT &= \sqrt{a^2 + \left(\frac{1}{2} a\sqrt{2}\right)^2} \\ &= \sqrt{a^2 + \frac{1}{2} a^2} = \sqrt{\frac{3}{2} a^2} \\ &= a \frac{\sqrt{3}}{\sqrt{2}} = \frac{1}{2} a\sqrt{6} \end{aligned}$$

$$\cos \alpha = \frac{BT}{FT} = \frac{\frac{1}{2} a\sqrt{2}}{\frac{1}{2} a\sqrt{6}} = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3}$$

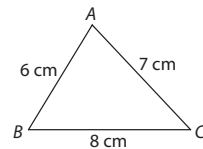


Jawaban: A

Bab 15 Trigonometri

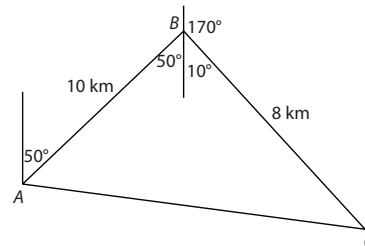
1. $a^2 = b^2 + c^2 - 2bc \cos A$

$$\begin{aligned} \cos A &= \frac{b^2 + c^2 - a^2}{2bc} = \frac{7^2 + 6^2 - 8^2}{2 \cdot 7 \cdot 6} \\ &= \frac{49 + 36 - 64}{2 \cdot 7 \cdot 6} = \frac{21}{2 \cdot 7 \cdot 6} = \frac{1}{4} \end{aligned}$$



Jawaban: B

2. Kapal berlayar dari A ke B sejauh 10 km dengan arah 050° . Kemudian dari B ke C sejauh 8 km dengan arah 170° .



$$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos 60^\circ$$

$$AC^2 = 10^2 + 8^2 - 2(10)(8) \cos 60^\circ$$

$$AC^2 = 100 + 64 - 160 \left(\frac{1}{2}\right)$$

$$AC^2 = 164 - 80 = 84$$

$$AC = \sqrt{84} = 2\sqrt{21}$$

Jadi, jarak pelabuhan C dari pelabuhan A adalah $2\sqrt{21}$ km.

Jawaban: D

3. $\cos 2x + \sin x - 1 = 0$

$$1 - 2\sin^2 x + \sin x - 1 = 0$$

$$-2\sin^2 x + \sin x = 0$$

$$\sin x (-2\sin x + 1) = 0$$

$$\sin x = 0, \text{ maka } x = 0, \pi, 2\pi$$

$$\sin x = \frac{1}{2}, \text{ maka } x = \frac{1}{6}\pi, \frac{5}{6}\pi$$

$$\text{Jadi, HP} = \left\{0, \frac{1}{6}\pi, \frac{5}{6}\pi, \pi, 2\pi\right\}$$

Jawaban: C

$$4. \quad \cos 2x + \sin x = 0$$

$$1 - 2\sin^2 x + \sin x = 0$$

$$-2\sin^2 x + \sin x + 1 = 0$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2}, \text{ maka } x = 210^\circ, 330^\circ$$

$$\sin x = 1, \text{ maka } x = 90^\circ$$

$$\text{HP} = \{90^\circ, 210^\circ, 330^\circ\}$$

Jawaban: D

5. Sudut A tumpul dengan $\sin A = \frac{4}{5} = \frac{y}{r}$ diperoleh $y = 4$ dan $r = 5$. Sehingga,

$$x = -\sqrt{r^2 - y^2} = -\sqrt{5^2 - 4^2} = -3 \quad \cos A = \frac{x}{r} = -\frac{3}{5}$$

Sudut B lancip dengan $\cos B = \frac{5}{13} = \frac{x}{r}$, diperoleh $x = 5$ dan $r = 13$.

Sehingga,

$$y = \sqrt{r^2 - x^2} = \sqrt{13^2 - 5^2} = 12$$

$$\sin B = \frac{y}{r} = \frac{12}{13}$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$= -\frac{3}{5} \cdot \frac{5}{13} + \frac{4}{5} \cdot \frac{12}{13} = -\frac{15}{65} + \frac{48}{65} = \frac{33}{65}$$

Jawaban: C

6. Sudut A di kuadran II dan $\sin A = \frac{3}{5} = \frac{y}{r}$, diperoleh

$$y = 3 \text{ dan } r = 5. \text{ Sehingga}$$

$$x = -\sqrt{r^2 - y^2} = -\sqrt{5^2 - 3^2} = -4$$

$$\cos A = \frac{x}{r} = -\frac{4}{5}$$

$$\sin 2A = 2 \sin A \cos A = 2 \cdot \frac{3}{5} \cdot -\frac{4}{5} = -\frac{24}{25}$$

Jawaban: E

7. $(A + B) = \frac{\pi}{2}$ dan $\sin A \sin B = \frac{1}{4}$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos \frac{\pi}{2} = \cos A \cos B - \sin A \sin B$$

$$0 = \cos A \cos B - \frac{1}{4}$$

$$\cos A \cos B = \frac{1}{4}$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Jawaban: C

$$8. \quad \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \frac{2 \sin \frac{1}{2}(5x + 3x) \cos \frac{1}{2}(5x - 3x)}{2 \cos \frac{1}{2}(5x + 3x) \cos \frac{1}{2}(5x - 3x)}$$

$$= \frac{2 \sin 4x \cos x}{2 \cos 4x \cos x} = \frac{\sin 4x}{\cos 4x} = \tan 4x$$

Jawaban: B

$$9. \quad \frac{\sin 100^\circ + \sin 20^\circ}{\cos 250^\circ + \cos 190^\circ} = \frac{2 \sin \left(\frac{100^\circ + 20^\circ}{2}\right) \cos \left(\frac{100^\circ - 20^\circ}{2}\right)}{2 \cos \left(\frac{250^\circ + 190^\circ}{2}\right) \cos \left(\frac{250^\circ - 190^\circ}{2}\right)}$$

$$= \frac{2 \sin 60^\circ \cos 40^\circ}{2 \cos 220^\circ \cos 30^\circ} = \frac{\sin 60^\circ \cos 40^\circ}{\cos 220^\circ \cos 30^\circ}$$

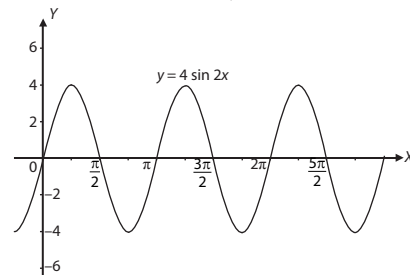
$$= \frac{\frac{1}{2} \sqrt{3} \cos 40^\circ}{\cos(180^\circ + 40^\circ) \frac{1}{2} \sqrt{3}}$$

$$= \frac{\cos 40^\circ}{-\cos 40^\circ} = -1$$

Jawaban: A

$$10. \quad y = 4 \sin 2x$$

Fungsi $y = 4 \sin 2x$ memiliki periode $\frac{2\pi}{2} = \pi$ dan memiliki amplitudo 4, yaitu minimum -4 dan maksimum 4. Sehingga grafik fungsi $y = 4 \sin 2x$ adalah



Jawaban: B

11. Jika diperhatikan grafik pada soal adalah grafik fungsi kosinus, bentuk umum fungsi kosinus adalah $f(x) = y = A \cos k(x + b)$

$$\text{Amplitudo, } A = \frac{1}{2}(-1 - (-1)) = -1$$

$$\text{Periode grafik, } p = \pi$$

$$k = \frac{2\pi}{p} = \frac{2\pi}{\pi} = 2$$

$$\text{Persamaan fungsinya adalah } y = -\cos 2(x + b)$$

$$\text{Grafik fungsi melalui titik } \left(0, -\frac{1}{2}\sqrt{3}\right)$$

$$-\frac{1}{2}\sqrt{3} = -\cos 2(0+b)$$

$$\frac{1}{2}\sqrt{3} = \cos 2b$$

$$\cos 30^\circ = \cos 2b$$

$$30^\circ = 2b$$

$$b = 15^\circ$$

Sehingga persamaan fungsinya menjadi:

$$y = -\cos 2(x + 15^\circ) = -\cos (2x + 30^\circ)$$

Jawaban: E

Bab 16 Limit Fungsi

1. Gunakan aturan L'Hopital

$$\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{2x - 5}{2x} = \frac{2(2) - 5}{2(2)} = -\frac{1}{4}$$

Jawaban: A

2. Gunakan aturan L'Hopital

$$\begin{aligned} \lim_{x \rightarrow 5} \frac{\sqrt{x+4} - \sqrt{14-x}}{x^2 - 2x - 15} &= \lim_{x \rightarrow 5} \frac{(x+4)^{\frac{1}{2}} - (14-x)^{\frac{1}{2}}}{x^2 - 2x - 15} \\ &= \lim_{x \rightarrow 5} \frac{\frac{1}{2}(x+4)^{-\frac{1}{2}} - \frac{1}{2}(-1)(14-x)^{-\frac{1}{2}}}{2x-2} \\ &= \frac{\frac{1}{2}(9)^{-\frac{1}{2}} - \frac{1}{2}(9)^{-\frac{1}{2}}}{2(5)-2} = \frac{\frac{1}{6} + \frac{1}{6}}{8} = \frac{1}{24} \end{aligned}$$

Jawaban: A

3. $\lim_{x \rightarrow \infty} (\sqrt{4x^2 + 4x - 3} - 2x + 5)$

$$\begin{aligned} &= \lim_{x \rightarrow \infty} (\sqrt{4x^2 + 4x - 3} - 2x) + 5 \\ &= \lim_{x \rightarrow \infty} (\sqrt{4x^2 + 4x - 3} - \sqrt{4x^2}) + 5 \\ &= \frac{4-0}{2\sqrt{4}} + 5 = \frac{4}{4} + 5 = 6 \end{aligned}$$

Jawaban: A

4. $\lim_{x \rightarrow 0} \frac{\sin 14x}{7x} = \frac{14}{7} = 2$

Jawaban: D

5. $\lim_{x \rightarrow 0} \frac{\cos 4x - 1}{x \tan 2x} = \lim_{x \rightarrow 0} \frac{-2 \sin^2 2x}{x \tan 2x}$

$$= \lim_{x \rightarrow 0} (-2) \cdot \frac{\sin 2x}{x} \cdot \frac{\sin 2x}{\tan 2x} = -2 \cdot 2 \cdot 1 = -4$$

Jawaban: E

Bab 17 Turunan Fungsi

1. $3x + 2y = 12 \Rightarrow y = \frac{1}{2}(12 - 3x)$

$$N = xy$$

$$= x \left(\frac{1}{2}(12 - 3x) \right) = 6x - \frac{3}{2}x^2$$

$$N' = 6 - 3x \Rightarrow 6 - 3x = 0 \Rightarrow x = 2$$

Sehingga,

$$y = \frac{1}{2}(12 - 3x) = \frac{1}{2}(12 - 3(2)) = \frac{1}{2}(12 - 6) = \frac{1}{2}(6) = 3$$

$$N = xy = (2)(3) = 6$$

Jawaban: C

2. $L_{\text{kotak}} = x^2 + 4xy = 432$

$$4xy = 432 - x^2$$

$$y = \frac{432 - x^2}{4x}$$

Sehingga,

$$V = x^2 y = x^2 \left(\frac{432 - x^2}{4x} \right) = 108x - \frac{1}{4}x^3$$

$$V' = 108 - \frac{3}{4}x^2$$

Volume kotak maksimum, maka $V' = 0$

$$\text{Sehingga, } \frac{3}{4}x^2 = 108$$

$$3x^2 = 432$$

$$x^2 = 144$$

$$x = 12$$

Jadi, panjang rusuk persegi adalah 12 cm.

Jawaban: D

3. $f(x) = y = \sin^3(5 - 4x)$

$$\text{Misalkan } u = 5 - 4x \Rightarrow \frac{du}{dx} = -4$$

$$y = \sin^3 u$$

$$\text{Misalkan } v = \sin u \Rightarrow \frac{dv}{du} = \cos u$$

$$y = v^3 \Rightarrow \frac{dy}{dv} = 3v^2$$

Sehingga,

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx} = 3v^2 \cdot \cos u \cdot (-4) \\ &= -12v^2 \cos u = -12(\sin u)^2 \cos u \\ &= -6 \sin u \sin 2u \\ &= -6 \sin(5 - 4x) \sin 2(5 - 4x) \\ &= -6 \sin(5 - 4x) \sin(10 - 8x) \end{aligned}$$

Jawaban: D

4. Absis -2, artinya $x = -2$

$$y = (-2)^3 + 2(-2) + 5$$

$$= -8 - 4 + 5 = -7$$

Titik yang dilalui garis singgung adalah $(-2, -7)$

$$y = x^3 + 2x + 5$$

$$y' = 3x^2 + 2$$

$$m = y'(-2) = 3(-2)^2 + 2$$

$$= 3(4) + 2 = 14$$

Persamaan garis singgung:

$$y - y_1 = m(x - x_1)$$

$$y + 7 = 14(x + 2)$$

$$y = 14x + 28 - 7$$

$$y = 14x + 21$$

Jawaban: E

Bab 18 Integral

$$\begin{aligned} 1. \quad \int_1^3 (3x-1)(x+1) dx &= \int_1^3 (3x^2 + 2x - 1) dx = x^3 + x^2 - x \Big|_1^3 \\ &= (3^3 - 1^3) + (3^2 - 1^2) - (3 - 1) \\ &= (27 - 1) + (9 - 1) - (3 - 1) \\ &= 26 + 8 - 2 = 32 \end{aligned}$$

Jawaban: C

$$2. \quad \text{Misalkan } x - 4 = u \Leftrightarrow x = u + 4$$

$$\begin{aligned} \int 3x(x-4)^5 dx &= 3 \int (u+4)u^5 du \\ &= 3 \int (u^6 + 4u^5) du \\ &= 3 \left(\frac{1}{7}u^7 + \frac{4}{6}u^6 \right) + C \\ &= 3u^6 \left(\frac{1}{7}u + \frac{2}{3} \right) + C \\ &= u^6 \left(\frac{3}{7}u + 2 \right) + C \\ &= (x-4)^6 \left(\frac{3}{7}(x-4) + 2 \right) + C \\ &= (x-4)^6 \frac{1}{7}(3x+2) + C \\ &= \frac{1}{7}(3x+2)(x-4)^6 + C \end{aligned}$$

Jawaban: B

$$\begin{aligned} 3. \quad \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^3 x \cos x dx &= \frac{1}{4} \sin^4 x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{4} \left(\sin^4 \left(\frac{\pi}{2} \right) - \sin^4 \left(\frac{\pi}{4} \right) \right) \\ &= \frac{1}{4} \left((1)^4 - \left(\frac{1}{2}\sqrt{2} \right)^4 \right) = \frac{1}{4} \left(1 - \frac{1}{4} \right) = \frac{1}{4} \left(\frac{3}{4} \right) = \frac{3}{16} \end{aligned}$$

Jawaban: C

$$4. \quad \int (\sin^2 2x \cos 2x) dx$$

$$\text{Misalkan } u = \sin 2x \Rightarrow dx = \frac{1}{2 \cos 2x} du$$

Sehingga,

$$\begin{aligned} \int (\sin^2 2x \cos 2x) dx &= \frac{1}{2} \int u^2 du \\ &= \frac{1}{2} \left(\frac{1}{3} u^3 \right) + C \\ &= \frac{1}{6} u^3 + C \\ &= \frac{1}{6} \sin^3 2x + C \end{aligned}$$

Jawaban: D

$$\begin{aligned} 5. \quad \int x \sqrt{2x^2+1} dx &= \frac{1}{4} \cdot \frac{1}{\frac{1}{2}+1} \cdot (\sqrt{2x^2+1})^{\frac{3}{2}} + C \\ &= \frac{1}{4} \cdot \frac{2}{3} (2x^2+1) (\sqrt{2x^2+1}) + C = \frac{1}{6} (2x^2+1) (\sqrt{2x^2+1}) + C \end{aligned}$$

Jawaban: E

$$6. \quad \text{Misalkan } u = x^2 - 3x - 5$$

$$\frac{du}{dx} = 2x - 3 \Leftrightarrow dx = \frac{du}{2x-3}$$

Sehingga,

$$\begin{aligned} \int \frac{6x-9}{\sqrt{x^2-3x-5}} dx &= \int \frac{3(2x-3)}{u^{\frac{1}{2}}} \frac{du}{2x-3} \\ &= 3 \int u^{-\frac{1}{2}} du \\ &= 3 \left(2u^{\frac{1}{2}} \right) + C \\ &= 6\sqrt{u} + C \\ &= 6\sqrt{x^2-3x-5} + C \end{aligned}$$

Jawaban: C

$$7. \quad \text{Titik potong garis } y = 2x \text{ dengan kurva } y = 8 - x^2, \text{ yaitu}$$

$$2x = 8 - x^2$$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

$$x = -4, x = 2$$

Titik potongnya adalah $(-4, 0)$ dan $(2, 0)$.

$$\begin{aligned} L &= \int_0^2 ((8-x^2)-2x) dx = \int_0^2 (8-2x-x^2) dx \\ &= \left(8x - x^2 - \frac{1}{3}x^3 \right) \Big|_0^2 = 8(2-0) - (2^2-0^2) - \frac{1}{3}(2^3-0^3) \\ &= 8(2) - (4) - \frac{1}{3}(8) = 16 - 4 - \frac{8}{3} = 12 - \frac{8}{3} = \frac{28}{3} = 9\frac{1}{3} \end{aligned}$$

Jadi, luas daerah arsiran adalah $9\frac{1}{3}$ satuan luas.

Jawaban: D

$$8. \quad \text{Titik potong } y = x^2 + 2x - 3 \text{ dengan } x = 2 \text{ adalah}$$

$$y = (2)^2 + 2(2) - 3$$

$$= 4 + 4 - 3 = 5$$

Titik potongnya adalah $(2, 5)$

$$\begin{aligned} L &= \int_0^2 (x^2 + 2x - 3) dx = \left(\frac{1}{3}x^3 + x^2 - 3x \right) \Big|_0^2 \\ &= \frac{1}{3}(2^3-0^3) + (2^2-0^2) - 3(2-0) \\ &= \frac{1}{3}(8) + (4) - 3(2) = \frac{8}{3} + 4 - 6 = \frac{2}{3} \end{aligned}$$

Jawaban: E

9. Titik potong pada sumbu-X, artinya $y = 0$

$$x^2 - 1 = 0$$

$$(x - 1)(x + 1) = 0$$

$$x = 1, x = -1$$

Titik potongnya adalah $(-1, 0)$ dan $(1, 0)$

$$\begin{aligned} V &= \pi \int_{-1}^1 (x^2 - 1)^2 dx = \pi \int_{-1}^1 (x^4 - 2x^2 + 1) dx \\ &= \pi \left(\frac{1}{5} x^5 - \frac{2}{3} x^3 + x \right) \Big|_{-1}^1 \\ &= \pi \left(\frac{1}{5} (1^5 - (-1)^5) - \frac{2}{3} (1^3 - (-1)^3) + (1 - (-1)) \right) \\ &= \pi \left(\frac{1}{5} (1+1) - \frac{2}{3} (1+1) + (1+1) \right) = \pi \left(\frac{2}{5} - \frac{4}{3} + 2 \right) \\ &= \pi \left(\frac{6-20+30}{15} \right) = \frac{16}{15} \pi \end{aligned}$$

Jawaban: C

Bab 19 Statistika

1.	Nilai	Frekuensi (f_i)	Nilai Tengah (x_i)	$f_i \cdot x_i$
	55-59	3	57	171
	60-64	5	62	310
	65-69	8	67	536
	70-74	16	72	1.152
	75-79	10	77	770
	80-84	6	82	492
	85-89	2	87	174
	Jumlah	50		3.605

$$\bar{x} = \frac{\sum f_i \cdot x_i}{\sum f_i} = \frac{3.605}{50} = 72,1$$

Jawaban: B

2. Berdasarkan histogram pada soal diperoleh:

Kelas modus = 24 - 28

$$T_b = 23,5$$

$$s_1 = 6$$

$$s_2 = 4$$

$$l = 5$$

Sehingga,

$$\begin{aligned} Mo &= T_b + \left(\frac{s_1}{s_1 + s_2} \right) \times l \\ &= 23,5 + \left(\frac{6}{6+4} \right) \times 5 \\ &= 23,5 + \frac{30}{10} \\ &= 23,5 + 3 = 26,5 \end{aligned}$$

Jawaban: D

3. $n = \sum f_i = 40$

Median = Me

Kelas Me berada pada data ke-

$$\frac{1}{2} \times n = \frac{1}{2} \times 40 = 20 \text{ (kelas ke-3)}$$

$$T_b = 56 - 0,5 = 55,5$$

$$l = 10$$

$$\begin{aligned} Me &= T_b + \left(\frac{\frac{1}{2}n - F_k}{f_{Me}} \right) l \\ &= 55,5 + \left(\frac{20 - 15}{12} \right) 10 \\ &= 55,5 + \frac{5}{12} \cdot 10 \\ &= 55,5 + 4,17 = 59,67 \end{aligned}$$

Jawaban: A

4. Kuartil bawah Q_1 terletak pada data ke $\frac{1}{4}(120) = 30$, yaitu kelas ke-4.

$$Q_1 = 59,5 + \left(\frac{30 - 15}{21} \right) (10) = 59,5 + 7,14 = 66,64$$

Jawaban: B

5. $n = \sum f_i = 40$

Kuartil atas = Q_3

Kelas Q_3 berada pada data ke-

$$\frac{3}{4} \times n = \frac{3}{4} \times 40 = 30 \text{ (kelas ke-4)}$$

$$T_b = 61 - 0,5 = 60,5$$

$$l = 20$$

$$\begin{aligned} Q_3 &= T_b + \left(\frac{\frac{3}{4}n - F_k}{f_{Q_3}} \right) l \\ &= 60,5 + \left(\frac{30 - 20}{15} \right) 20 \\ &= 60,5 + \frac{10}{15} \cdot 20 \\ &= 60,5 + 13,33 = 73,83 \end{aligned}$$

Jawaban: E

Bab 20 Peluang

1. Akan dibuat nomor pegawai yang terdiri atas tiga angka.

I	II	III
---	----	-----

Kotak I tidak boleh diisi angka 0, sehingga ada 9 cara.

Kotak II dapat diisi dengan 10 angka, sehingga ada 10 cara.

Kotak III dapat diisi dengan angka ganjil yaitu 1, 3, 5, 7, atau 9, sehingga ada 5 cara.

Jadi, banyaknya cara adalah $9 \times 10 \times 5 = 450$

Jawaban: C

2. Susunan pengurus yang mungkin terjadi merupakan permutasi 3 dari 7 orang calon, sehingga

$$P(7, 3) = \frac{7!}{(7-3)!} = \frac{7 \times 6 \times 5 \times 4!}{4!} = 210$$

Jawaban: E

3. 4 soal pertama wajib dikerjakan, berarti ada 4 soal yang harus dipilih dari 6 soal sisa. Susunan soal tersebut merupakan kombinasi 4 dari 6.

$$C(6, 4) = \frac{6!}{(6-4)!4!} = \frac{6 \times 5 \times 4!}{2 \times 1 \times 4!} = 15$$

Jawaban: B

4. Misalkan

$$A = \{\text{kejadian munculnya mata dadu berjumlah 7}\} \\ = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$$

$$n(A) = 6$$

$$B = \{\text{kejadian munculnya mata dadu berjumlah 10}\} \\ = \{(4, 6), (5, 5), (6, 4)\}$$

$$n(B) = 3$$

$$S = \{\text{ruang sampel dua buah dadu}\}, n(S) = 36$$

Kejadian munculnya mata dadu berjumlah 7 tidak terjadi secara bersamaan dengan kejadian munculnya

mata dadu berjumlah 10. Dengan demikian A dan B merupakan kejadian saling lepas.

$$P(A \text{ atau } B) = P(A) + P(B) = \frac{6}{36} + \frac{3}{36} = \frac{9}{36}$$

Jawaban: D

$$5. \quad C(4, 2) = \frac{4!}{(4-2)!2!} = \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} = 6$$

$$C(52, 2) = \frac{52!}{(52-2)!2!} = \frac{52 \times 51 \times 50!}{2 \times 1 \times 50!} = 1.326$$

Peluang terambil kartu King

$$\frac{C(4, 2)}{C(52, 2)} = \frac{6}{1.326} = \frac{1}{221}$$

Jawaban: A

6. Peluang membuat gol, $P(G) = \frac{3}{5}$

$$\text{Peluang tidak gol, } P(G') = \frac{2}{5}$$

Peluang membuat 2 gol dari 3 tendangan:

$$\frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} = \frac{18}{125}$$

Jadi, peluang Ali untuk membuat 2 gol adalah $\frac{18}{125}$.

Jawaban: A

PEMBAHASAN DAN KUNCI JAWABAN PEMAHAMAN PAKET 1

MATEMATIKA IPA

$$1. \frac{49^{\frac{3}{2}} 8^{\frac{5}{3}}}{(7 \cdot 2)^3} = \frac{(49)^{\frac{3}{2}} \cdot (8)^{\frac{5}{3}}}{(7)^3 \cdot (2)^3} = \frac{(7^2)^{\frac{3}{2}} \cdot (2^3)^{\frac{5}{3}}}{7^3 \cdot 2^3}$$

$$= \frac{7^{2 \cdot \frac{3}{2}} \cdot 2^{3 \cdot \frac{5}{3}}}{7^3 \cdot 2^3} = \frac{7^3 \cdot 2^5}{7^3 \cdot 2^3}$$

$$= 7^{3-3} \cdot 2^{5-3} = 2^2 = 4$$

Jawaban: B

$$2. \frac{\sqrt{5}-3\sqrt{3}}{(\sqrt{5}+\sqrt{3})(\sqrt{3}-\sqrt{5})} = \frac{\sqrt{5}-3\sqrt{3}}{\sqrt{15}-5+3-\sqrt{15}}$$

$$= \frac{\sqrt{5}-3\sqrt{3}}{-2}$$

$$= \frac{1}{2}(3\sqrt{3}-\sqrt{5})$$

Jawaban: C

$$3. \frac{\frac{1}{2} {}^c \log b^2 \cdot 3 {}^b \log c + {}^{c^{\frac{1}{2}}} \log c^2}{3 {}^b \log(b^2) - {}^{b^{\frac{1}{2}}} \log b}$$

$$= \frac{\frac{1}{2} \cdot 2 \cdot 3 {}^c \log b \cdot {}^b \log c + 2 \cdot 2 {}^c \log c}{3 \cdot 2 \cdot {}^b \log b - 2 \cdot {}^b \log b}$$

$$= \frac{3+4}{6-2} = \frac{7}{4}$$

Jawaban: D

$$4. 2(4^x) - 5(2^x) + 2 > 0$$

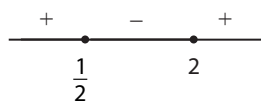
$$2(2^x)^2 - 5(2^x) + 2 > 0$$

Misalkan $2^x = y$, maka pertidaksamaannya menjadi:

$$2y^2 - 5y + 2 > 0$$

$$(2y-1)(y-2) > 0$$

Pembuat nol fungsi: $y = \frac{1}{2}$ dan $y = 2$



$$y < \frac{1}{2} \text{ atau } y > 2$$

$$2^x < \frac{1}{2} \quad 2^x > 2$$

$$2^x < 2^{-1} \quad 2^x > 2^1$$

$$x < -1 \quad x > 1$$

Jadi, nilai x yang memenuhi adalah $x < -1$ atau $x > 1$.

Jawaban: A

$$5. (f \circ g)(x) = f(g(x)) = x^2 + 6x + 7$$

$$f(x) = x + 3 \Rightarrow f(g(x)) = g(x) + 3$$

$$x^2 + 6x + 7 = g(x) + 3$$

$$x^2 + 6x + 7 - 3 = g(x)$$

$$x^2 + 6x + 4 = g(x)$$

$$g(x) = x^2 + 6x + 4$$

Jawaban: D

$$6. f(x) = \frac{2x-1}{x+1}, x \neq -1 \text{ dan } g(x) = 4x - 3$$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{2x-1}{x+1}\right)$$

$$= 4\left(\frac{2x-1}{x+1}\right) - 3$$

$$= \frac{8x-4}{x+1} - 3\left(\frac{x+1}{x+1}\right)$$

$$= \frac{8x-4}{x+1} - \frac{3x+3}{x+1}$$

$$= \frac{5x-7}{x+1}, x \neq -1$$

$$\text{Jika } f(x) = \frac{ax+b}{cx+d}, x \neq -\frac{d}{c}, \text{ maka } f^{-1}(x) = \frac{-dx+b}{cx-a}.$$

$$\text{Karena } (g \circ f)(x) = \frac{5x-7}{x+1}, x \neq -1, \text{ maka}$$

$$(g \circ f)^{-1}(x) = \frac{-x-7}{x-5} \times \frac{-1}{-1} = \frac{x+7}{5-x}, x \neq 5$$

Jawaban: B

$$7. \text{PK: } x^2 + ax - 4 = 0$$

Akar-akar: p dan q

$$p + q = -a$$

$$pq = -4$$

$$(p-q)^2 = 4(2a)$$

$$p^2 + q^2 - 2pq = 8a$$

$$(p+q)^2 - 2pq - 2pq = 8a$$

$$(p+q)^2 - 4pq = 8a$$

$$(-a)^2 - 4(-4) = 8a$$

$$a^2 + 16 - 8a = 0$$

$$(a-4)^2 = 0$$

$$a - 4 = 0$$

$$a = 4$$

Jawaban: E

$$8. x^2 + 2x + 4 = 0, \text{ akar-akarnya } a \text{ dan } b.$$

$$a + b = -2$$

$$ab = 4$$

Persamaan kuadrat baru yang akar-akarnya $3a + 2$ dan $3b + 2$.

Misal:

$$c = 3a + 2$$

$$d = 3b + 2$$

$$\begin{aligned} c + d &= (3a + 2) + (3b + 2) = 3a + 3b + 4 \\ &= 3(a + b) + 4 = 3(-2) + 4 \\ &= -6 + 4 = -2 \end{aligned}$$

$$\begin{aligned} cd &= (3a + 2)(3b + 2) = 9ab + 6a + 6b + 4 \\ &= 9(ab) + 6(a + b) + 4 \\ &= 9(4) + 6(-2) + 4 \\ &= 36 - 12 + 4 = 28 \end{aligned}$$

PK baru:

$$\begin{aligned} x^2 - (c + d)x + cd &= 0 \\ x^2 - (-2)x + 28 &= 0 \\ x^2 + 2x + 28 &= 0 \end{aligned}$$

Jawaban: D

9. Semua warga negara yang berumur di atas 17 tahun memiliki hak pilih. Semua mahasiswa S-1 berumur di atas 17 tahun. Jadi, semua mahasiswa S-1 memiliki hak pilih.

Jawaban: A

10. Fungsi $y = x^2 + cx + d$

Dengan, $a = 1$, $b = c$, dan $\bar{c} = d$

Titik puncak: $\left(-\frac{b}{2a}, -\frac{D}{4a}\right) = (1, 2)$, sehingga

$$-\frac{b}{2a} = -\frac{c}{2(1)} = 1 \Leftrightarrow c = -2$$

Grafik fungsinya $y = x^2 - 2x + d$

Substitusikan (1, 2) ke fungsi.

$$2 = (1)^2 - 2(1) + d$$

$$2 = 1 - 2 + d$$

$$2 = -1 + d$$

$$d = 3$$

$$\text{Jadi, } c + d = -2 + 3 = 1$$

Jawaban: C

11. Misalkan umur Pak Andi = A, umur Bu Andi = B, dan umur Amira = C

$$A = C + 28 \Rightarrow C = A - 28$$

$$B = A - 6$$

$$A + B + C = 119$$

$$B + C = 119 - A$$

Kita ubah B dan C dalam bentuk A untuk memperoleh nilai A.

$$A + B + C = 119$$

$$A + (A - 6) + (A - 28) = 119$$

$$3A - 34 = 119$$

$$A = \frac{153}{3} = 51$$

$$B + C = 119 - A = 119 - 51 = 68$$

Jadi, jumlah umur Amira dan Bu Andi adalah 68 tahun.

Jawaban: C

12. Misalkan harga pulpen = a, buku tulis = b, dan pensil = c

$$\text{Ani} \equiv 3a + b + 2c = 22.000 \quad \dots(1)$$

$$\text{Beri} \equiv 2a + 3b + c = 28.000 \quad \dots(2)$$

$$\text{Cindi} \equiv a + 2b + 3c = 22.000 \quad \dots(3)$$

Dari persamaan (1) dan (2) diperoleh:

$$\begin{array}{r} 3a + b + 2c = 22.000 \quad \times 3 \\ 2a + 3b + c = 28.000 \quad \times 1 \\ \hline 9a + 3b + 6c = 66.000 \\ 2a + 3b + c = 28.000 \\ \hline 7a + 5c = 38.000 \quad \dots(4) \end{array}$$

Dari persamaan (2) dan (3) diperoleh:

$$\begin{array}{r} 2a + 3b + c = 28.000 \quad \times 2 \\ a + 2b + 3c = 22.000 \quad \times 3 \\ \hline 4a + 6b + 2c = 56.000 \\ 3a + 6b + 9c = 66.000 \\ \hline a - 7c = -10.000 \quad \dots(5) \end{array}$$

Dari persamaan (4) dan (5) diperoleh:

$$\begin{array}{r} 7a + 5c = 38.000 \quad \times 1 \\ a - 7c = -10.000 \quad \times 7 \\ \hline 7a + 5c = 38.000 \\ 7a - 49c = -70.000 \\ \hline 54c = 108.000 \\ c = 2.000 \end{array}$$

Substitusikan $c = 2.000$ ke persamaan (5) diperoleh

$$a - 7(2.000) = -10.000$$

$$a - 14.000 = -10.000$$

$$a = -10.000 + 14.000$$

$$a = 4.000$$

Substitusikan $a = 4.000$ dan $c = 2.000$ ke persamaan (1) diperoleh

$$3(4.000) + b + 2(2.000) = 22.000$$

$$12.000 + b + 4.000 = 22.000$$

$$b + 16.000 = 22.000$$

$$b = 22.000 - 16.000$$

$$b = 6.000$$

$$\text{Deni} \equiv 2a + b + c$$

$$= 2(4.000) + 6.000 + 2.000 = 16.000$$

Jadi, Deni harus membayar sebesar Rp16.000,00.

Jawaban: B

$$13. AB = C \Rightarrow \begin{pmatrix} x-5 & 4 \\ -5 & 2 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 2 & y-1 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -16 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 4x-20+8 & -x+5+4y-4 \\ -20+4 & 5+2y-2 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -16 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 4x-12 & 4y-x+1 \\ -16 & 2y+3 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -16 & 5 \end{pmatrix}$$

Diperoleh $4x - 12 = 0 \Rightarrow x = 3$

$2y + 3 = 5 \Rightarrow y = 1$

Jadi, $x : y = 3 : 1$

Jawaban: A

14. Diketahui matriks $A = \begin{pmatrix} 2 & -3 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} -4 & 2 \\ 1 & 2 \end{pmatrix}$, dan

$$C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}.$$

$$A + (B \times C) = \begin{pmatrix} 2 & -3 \\ -1 & 0 \end{pmatrix} + \left(\begin{pmatrix} -4 & 2 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \right)$$

$$= \begin{pmatrix} 2 & -3 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 4+2 & 0+(-2) \\ -1+2 & 0+(-2) \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -3 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 6 & -2 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 8 & -5 \\ 0 & -2 \end{pmatrix}$$

$$\det(A + (B \times C)) = \begin{vmatrix} 8 & -5 \\ 0 & -2 \end{vmatrix}$$

$$= (8)(-2) - (0)(-5)$$

$$= -16$$

Jawaban: A

15. $U_6 = a + 5b = 43$

$$U_{12} = a + 11b = 85$$

$$a + 5b + 6b = 85$$

$$43 + 6b = 85$$

$$6b = 85 - 43 = 42$$

$$b = 7$$

$$a + 5(7) = 43$$

$$a + 35 = 43$$

$$a = 43 - 35 = 8$$

Sehingga,

$$S_{25} = \frac{25}{2} (2(8) + (25-1)7)$$

$$= \frac{25}{2} (16 + (24)7) = \frac{25}{2} (16 + 168)$$

$$= \frac{25}{2} (184) = 25(92) = 2.300$$

Jawaban: B

16. $U_9 = ar^8 = (ar^4)r^4 = U_5 \cdot r^4 = \frac{1}{3} \left(\frac{1}{3} \right)^4 = \left(\frac{1}{3} \right)^5 = \frac{1}{243}$

Jawaban: E

17. Misalkan pengambilan bulan pertama U_1

$$U_1 = a = 2.000.000$$

$$U_2 = a + b = 1.925.000$$

$$U_3 = a + 2b = 1.850.000$$

$$b = U_2 - U_1$$

$$= 1.925.000 - 2.000.000 = -75.000$$

Karena $a = 2.000.000$ dan $b = -75.000$, sehingga

$$S_{12} = \frac{12}{2} (2(2.000.000) + (12-1)(-75.000))$$

$$= 6(4.000.000 + (11)(-75.000))$$

$$= 6(4.000.000 + (-825.000))$$

$$= 6(3.175.000) = 19.050.000$$

Jadi, jumlah seluruh uang yang telah diambil selama 12 bulan pertama adalah Rp19.050.000,00.

Jawaban: B

18. $f(x) = P(x) \cdot H(x) + S$

$$f(x) = (x^2 + 2x - 3) \cdot (x + h) + (3x - 4)$$

$$f(x) = (x^2 - x - 2) \cdot (x + k) + (2x + 3)$$

Faktor dari : $(x^2 + 2x - 3) = 0$

$$(x - 1)(x + 3) = 0$$

$$x_1 = 1, x_2 = -3$$

$$S = f(1) = 3(1) - 4 = -1$$

$$S = f(-3) = 3(-3) - 4 = -13$$

$$f(x) = (x^2 - x - 2) \cdot (x + k) + (2x + 3)$$

$$f(1) = (1 - 1 - 2) \cdot (1 + k) + (2 + 3)$$

$$-1 = -2(1 + k) + 5$$

$$-6 = -2 - 2k$$

$$2k = 4$$

$$k = 2$$

$$f(x) = (x^2 - x - 2) \cdot (x + 2) + (2x + 3)$$

$$f(x) = x^3 + 2x^2 - x^2 - 2x - 4 + 2x + 3$$

$$f(x) = x^3 + x^2 - 2x - 1$$

Jawaban: B

19. $\lim_{x \rightarrow 2} \frac{4 - x^2}{3 - \sqrt{x^2 + 5}} = \lim_{x \rightarrow 2} \frac{4 - x^2}{3 - \sqrt{x^2 + 5}} \cdot \frac{3 + \sqrt{x^2 + 5}}{3 + \sqrt{x^2 + 5}}$

$$= \lim_{x \rightarrow 2} \frac{(4 - x^2)(3 + \sqrt{x^2 + 5})}{(3)^2 - (\sqrt{x^2 + 5})^2}$$

$$= \lim_{x \rightarrow 2} \frac{(4 - x^2)(3 + \sqrt{x^2 + 5})}{9 - (x^2 + 5)}$$

$$= \lim_{x \rightarrow 2} \frac{(4 - x^2)(3 + \sqrt{x^2 + 5})}{4 - x^2}$$

$$= \lim_{x \rightarrow 2} (3 + \sqrt{x^2 + 5}) = 3 + \sqrt{2^2 + 5}$$

$$= 3 + \sqrt{4 + 5} = 3 + \sqrt{9} = 3 + 3 = 6$$

Jawaban: B

$$\begin{aligned}
20. \lim_{x \rightarrow \infty} \{ (2x+1) - \sqrt{4x^2 - 3x + 6} \} \\
&= \lim_{x \rightarrow \infty} \{ 2x - \sqrt{4x^2 - 3x + 6} \} + 1 \\
&= \lim_{x \rightarrow \infty} \{ \sqrt{4x^2} - \sqrt{4x^2 - 3x + 6} \} + 1 \\
&= \frac{0 - (-3)}{2\sqrt{4}} + 1 \\
&= \frac{3}{4} + 1 = \frac{7}{4}
\end{aligned}$$

Jawaban: C

21. Gradien persamaan garis singgungnya diperoleh:

$$m = y' = 3x^2 - 2$$

$$m_{x=1} = 3(1)^2 - 2 = 3 - 2 = 1$$

Persamaan garis singgungnya:

$$y - 0 = 1(x - 1)$$

$$y = x - 1$$

Substitusikan $x = 3$

$$y = 3 - 1 = 2$$

Jadi, garis singgung akan memotong garis $x = 3$ di titik $(3, 2)$.

Jawaban: B

22. Misalkan $u = x^2 - 6x + 1$

$$\frac{du}{dx} = 2x - 6 = 2(x - 3)$$

$$dx = \frac{du}{2(x-3)}$$

Sehingga,

$$\begin{aligned}
&\int (x-3)(x^2-6x+1)^{-3} dx \\
&= \int (x-3)u^{-3} \frac{du}{2(x-3)} = \frac{1}{2} \int u^{-3} du \\
&= \frac{1}{2} \cdot \left(-\frac{1}{2} u^{-2} \right) + C = -\frac{1}{4} u^{-2} + C \\
&= -\frac{1}{4} (x^2 - 6x + 1)^{-2} + C
\end{aligned}$$

Jawaban: D

$$\begin{aligned}
23. \int_1^2 (4x^2 - x + 5) dx &= \left[\frac{4}{3}x^3 - \frac{1}{2}x^2 + 5x \right]_1^2 \\
&= \left[\frac{4}{3}(2)^3 - \frac{1}{2}(2)^2 + 5(2) \right] - \left[\frac{4}{3}(1)^3 - \frac{1}{2}(1)^2 + 5(1) \right] \\
&= \left[\frac{4}{3}(8) - 2 + 10 \right] - \left[\frac{4}{3} - \frac{1}{2} + 5 \right] \\
&= \frac{32}{3} + 8 - \frac{4}{3} + \frac{1}{2} - 5 \\
&= \frac{28}{3} + \frac{1}{2} + 3 = \frac{56+3+18}{6} = \frac{77}{6}
\end{aligned}$$

Jawaban: E

24. Fungsi $y = x^3 - 3x + 4$

$$y' = 3x^2 - 3$$

$$\text{Stasioner} \Leftrightarrow y' = 0$$

$$y' = 3x^2 - 3 = 0$$

$$x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

Diperoleh $x_1 = -1$ dan $x_2 = 1$

Jika $y'' < 0$, maka fungsi naik

Jika $y'' > 0$, maka fungsi turun

$$y'' = 6x$$

Untuk $x_1 = -1$,

$y'' = 6(-1) = -6 < 0$, maka fungsi naik dan terjadi titik balik maksimum pada $x = -1$

Substitusikan $x = -1$ ke fungsi y , diperoleh:

$$y = (-1)^3 - 3(-1) + 4$$

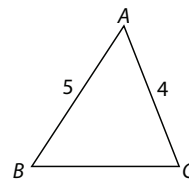
$$y = -1 + 3 + 4$$

$$y = 2 + 4 = 6$$

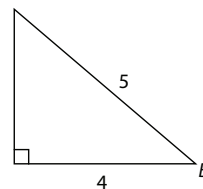
Jadi, koordinat titik baliknya adalah $(-1, 6)$.

Jawaban: A

25. Segitiga lancip ABC dengan $AC = 4$ cm dan $AB = 5$ cm.



$$\cos B = \frac{4}{5} = \frac{sa}{mi}$$



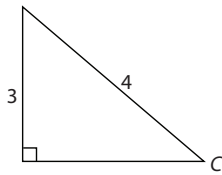
$$\begin{aligned}
de &= \sqrt{mi^2 - sa^2} = \sqrt{5^2 - 4^2} \\
&= \sqrt{25 - 16} = \sqrt{9} = 3
\end{aligned}$$

$$\text{Sehingga, } \sin B = \frac{de}{mi} = \frac{3}{5}$$

Aturan sinus

$$\begin{aligned}
\frac{b}{\sin B} &= \frac{c}{\sin C} \\
\frac{4}{\frac{3}{5}} &= \frac{5}{\sin C}
\end{aligned}$$

$$\sin C = \frac{5}{4} \times \frac{3}{5} = \frac{3}{4} = \frac{de}{mi}$$



$$sa = \sqrt{mi^2 - de^2} = \sqrt{4^2 - 3^2}$$

$$= \sqrt{16 - 9} = \sqrt{7}$$

$$\cos C = \frac{sa}{mi} = \frac{\sqrt{7}}{4} = \frac{1}{4}\sqrt{7}$$

Jawaban: B

$$26. \frac{\cos 10^\circ}{\cos 40^\circ \cos 50^\circ} = \frac{\cos 10^\circ}{\frac{1}{2} [\cos(40^\circ + 50^\circ) + \cos(50^\circ - 40^\circ)]}$$

$$= \frac{2 \cos 10^\circ}{\cos 90^\circ + \cos 10^\circ} = \frac{2 \cos 10^\circ}{\cos 10^\circ} = 2$$

Jawaban: B

$$27. \cos 2x + 3 \sin x = 2$$

$$1 - 2 \sin^2 x + 3 \sin x - 2 = 0$$

$$-2 \sin^2 x + 3 \sin x - 1 = 0$$

$$2 \sin^2 x - 3 \sin x + 1 = 0$$

$$(2 \sin x - 1)(\sin x - 1) = 0$$

$$\sin x = \frac{1}{2} \text{ atau } \sin x = 1$$

$$\text{Untuk } \sin x = \frac{1}{2}$$

$$\sin x = \frac{1}{2}$$

$$\sin x = \sin 30^\circ$$

$$x = 30^\circ + k \cdot 360^\circ$$

$$k = 0 \Rightarrow x = 30^\circ + 0 \cdot 360^\circ = 30^\circ$$

Atau

$$x = (180^\circ - 30^\circ) + k \cdot 360^\circ$$

$$x = 150^\circ + k \cdot 360^\circ$$

$$k = 0 \Rightarrow x = 150^\circ + 0 \cdot 360^\circ = 150^\circ$$

$$\text{Untuk } \sin x = 1$$

$$\sin x = 1$$

$$\sin x = \sin 90^\circ$$

$$x = 90^\circ + k \cdot 360^\circ$$

$$k = 0 \Rightarrow x = 90^\circ + 0 \cdot 360^\circ = 90^\circ$$

atau

$$x = (180^\circ - 90^\circ) + k \cdot 360^\circ$$

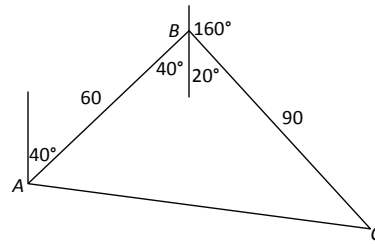
$$x = 90^\circ + k \cdot 360^\circ$$

$$k = 0 \Rightarrow x = 90^\circ + 0 \cdot 360^\circ = 90^\circ$$

$$HP = \{30^\circ, 90^\circ, 150^\circ\}$$

Jawaban: D

28. Kapal berlayar dari A ke B sejauh 60 mil dengan arah 040° . Kemudian dari B ke C sejauh 90 mil dengan arah 160° .



$$AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos 60^\circ$$

$$AC^2 = 60^2 + 90^2 - 2 \cdot 60 \cdot 90 \cdot \frac{1}{2}$$

$$AC^2 = 3.600 + 8.100 - 5.400$$

$$AC^2 = 11.700 - 5.400$$

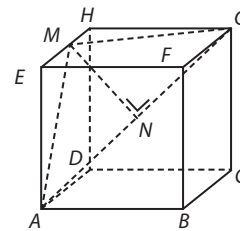
$$AC^2 = 6.300$$

$$AC = \sqrt{6.300} = 30\sqrt{7}$$

Jadi, jarak terdekat dari pelabuhan A ke C adalah $30\sqrt{7}$ mil.

Jawaban: C

29. Kubus $ABCD.EFGH$ dengan rusuk = 8 cm, M titik tengah EH .



$$AG = \text{diagonal ruang} = 8\sqrt{3}$$

Perhatikan $\triangle GHM$

$$GM = \sqrt{GH^2 + HM^2}$$

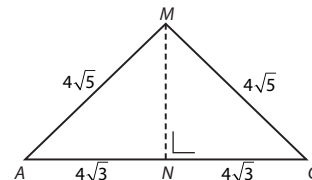
$$= \sqrt{8^2 + 4^2} = \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5}$$

Perhatikan $\triangle AEM$

$$AM = \sqrt{AE^2 + EM^2}$$

$$= \sqrt{8^2 + 4^2} = \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5}$$

Perhatikan $\triangle AGM$



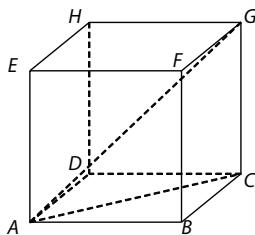
$$MN = \sqrt{(4\sqrt{5})^2 - (4\sqrt{3})^2}$$

$$= \sqrt{80 - 48} = \sqrt{32} = 4\sqrt{2}$$

Jadi, jarak titik M ke AG adalah $4\sqrt{2}$ cm.

Jawaban: D

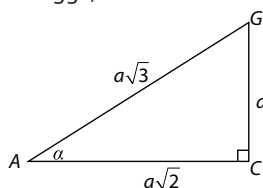
30. Perhatikan kubus $ABCD.EFGH$ berikut ini.



Garis AG merupakan diagonal ruang, sehingga $AG = a\sqrt{3}$.

Sudut antara diagonal AG dengan bidang alas $ABCD$ sama dengan sudut antara garis AG dengan garis AC , yaitu α , dengan $AC = a\sqrt{2}$.

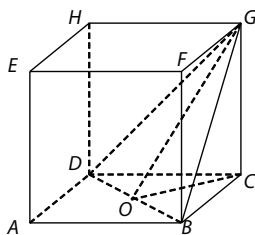
Sehingga,



$$\sin \alpha = \frac{GC}{AG} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3}$$

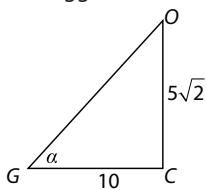
Jawaban: C

31. Perhatikan kubus $ABCD.EFGH$ berikut ini.



$$\angle(GC, BDG) = \angle(GC, GO) = \alpha$$

Sehingga,



$$\tan \alpha = \frac{OC}{GC} = \frac{5\sqrt{2}}{10} = \frac{1}{2}\sqrt{2}$$

Jawaban: C

32. $\vec{a} = 5\vec{i} + 6\vec{j} + \vec{k}$ dan $\vec{b} = \vec{i} - 2\vec{j} - 2\vec{k}$

Proyeksi orthogonal vektor $\vec{a}$ pada $\vec{b}$

$$\vec{a}_b = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \cdot \vec{b}$$

$$\vec{a}_b = \frac{5-12-2}{(\sqrt{1+4+4})^2} \cdot (\vec{i} - 2\vec{j} - 2\vec{k})$$

$$\vec{a}_b = \frac{-9}{9} \cdot (\vec{i} - 2\vec{j} - 2\vec{k}) = -\vec{i} + 2\vec{j} + 2\vec{k}$$

Jawaban: D

33. $x^2 + y^2 - 2x + 4y + 1 = 0$

$$x^2 - 2x + y^2 + 4y = -1$$

$$x^2 - 2x + 1 + y^2 + 4y + 4 = -1 + 1 + 4$$

$$(x-1)^2 + (y+2)^2 = 4$$

$$P(1, -2) = (a, b)$$

Sehingga,

$$2a + b = 2(1) + (-2) = 2 - 2 = 0$$

Jawaban: C

34. Lingkaran yang berpusat di $(1, -10)$ dan menyinggung garis $3x - y\sqrt{3} - 3 = 0$.

Jarak titik $(1, -10)$ ke garis $3x - y\sqrt{3} - 3 = 0$ adalah

$$d = \frac{|3(1) + (-\sqrt{3})(-10) + (-3)|}{\sqrt{3^2 + (-\sqrt{3})^2}} = \frac{|3 + 10\sqrt{3} - 3|}{\sqrt{9+3}} = \frac{|10\sqrt{3}|}{2\sqrt{3}} = 5$$

Sehingga persamaan lingkarannya adalah

$$(x-1)^2 + (y+10)^2 = 5^2$$

$$x^2 - 2x + 1 + y^2 + 20y + 100 = 25$$

$$x^2 + y^2 - 2x + 20y + 76 = 0$$

Jawaban: A

35. Pencermian terhadap garis $y = x$

$$(x, y) \rightarrow (x', y') = (y, x)$$

Dengan:

$$x' = y \Leftrightarrow y = x' \quad \dots (1)$$

$$y' = x \Leftrightarrow x = y' \quad \dots (2)$$

Substitusikan (1) dan (2) ke persamaan garis diperoleh:

$$y = 2x + 2$$

$$x' = 2y' + 2$$

$$x' - 2 = 2y'$$

$$y' = \frac{x' - 2}{2}$$

$$y' = \frac{1}{2}x' - 1$$

$$y = \frac{1}{2}x - 1$$

Jawaban: C

36. Data dengan frekuensi terbanyak terdapat di kelas dengan interval 10,5–15,5.

$$T_b = 10,5$$

$$I = 15,5 - 10,5 = 5$$

$$s_1 = 14 - 8 = 6$$

$$s_2 = 14 - 12 = 2$$

$$\begin{aligned} Mo &= T_b + \frac{s_1}{s_1 + s_2} \cdot I \\ &= 10,5 + \frac{6}{6+2} \cdot 5 \\ &= 10,5 + \frac{30}{8} \\ &= 10,5 + 3,75 = 14,25 \end{aligned}$$

Jawaban: A

37. $n = \sum f_i = 40$

Kuartil atas = Q_3

Kelas Q_3 berada pada data ke-

$$\frac{3}{4} \times n = \frac{3}{4} \times 40 = 30 \text{ (kelas ke-4)}$$

$$T_b = 70 - 0,5 = 69,5$$

Panjang kelas, $I = 10$

$$\begin{aligned} Q_3 &= T_b + \left(\frac{\frac{3}{4}n - F_k}{f_{Q_3}} \right) I \\ &= 69,5 + \left(\frac{30 - 23}{8} \right) 10 \\ &= 69,5 + \frac{70}{8} \\ &= 69,5 + 8,75 = 78,25 \end{aligned}$$

Jawaban: C

38. Angka-angka 0, 1, 2, 3, 4, 5, 6 akan disusun menjadi suatu bilangan lebih dari 320 yang terdiri atas 3 angka.

Kasus I: ratusan > 3

Ratusan: 4, 5, 6 = 3 cara

Puluhan: 6 cara

Satuan: 5 cara

Sehingga, $3 \times 6 \times 5 = 90$ cara

Kasus II: ratusan 3, puluhan > 2

Ratusan: 3 = 1 cara

Puluhan: 4, 5, 6 = 3 cara

Satuan: 5 cara

Sehingga, $1 \times 3 \times 5 = 15$ cara

Kasus III: ratusan 3, puluhan 2, satuan > 0

Ratusan: 3 = 1 cara

Puluhan: 2 = 1 cara

Satuan: 1, 4, 5, 6 = 4 cara

Sehingga, $1 \times 1 \times 4 = 4$ cara

Total = $90 + 15 + 4 = 109$

Jadi, banyak bilangan yang dapat disusun lebih dari 320 adalah 109 cara.

Jawaban: D

39. Terdapat 10 soal, 3 soal wajib dikerjakan, tersisa 5 soal lagi.

Akan dipilih 5 soal dari 7 soal tersedia sehingga

$$\begin{aligned} {}_7C_5 &= \frac{7!}{(7-5)!5!} = \frac{7!}{2!5!} \\ &= \frac{7 \times 6 \times 5!}{2 \times 1 \times 5!} = 7 \times 3 = 21 \end{aligned}$$

Jadi, seorang peserta ujian memilih soal yang dikerjakan adalah 21 cara.

Jawaban: B

40. Misalkan S = semua jenis ikan di kolam,

$$n(S) = 21 + 12 + 27 = 60$$

M = kejadian terpancingnya satu ikan mas

$$n(M) = 12$$

$$P(M) = \frac{n(M)}{n(S)} = \frac{12}{60} = \frac{1}{5}$$

Jadi, peluang Pak Amir mendapatkan ikan mas untuk satu kali memancing adalah $\frac{1}{5}$.

Jawaban: B

PEMBAHASAN DAN KUNCI JAWABAN PEMAHAMAN PAKET 2

MATEMATIKA IPA

$$1. \frac{7x^3y^{-4}z^{-6}}{84x^{-7}y^{-1}z^{-4}} = \frac{7}{84}x^{3-(-7)}y^{-4-(-1)}z^{-6-(-4)}$$

$$= \frac{1}{12}x^{10}y^{-3}z^{-2} = \frac{x^{10}}{12y^3z^2}$$

Jawaban: E

$$2. \frac{\sqrt{2}-2\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{2}-2\sqrt{3}}{\sqrt{2}-\sqrt{3}} \times \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}}$$

$$= \frac{(\sqrt{2}-2\sqrt{3})(\sqrt{2}+\sqrt{3})}{(\sqrt{2})^2-(\sqrt{3})^2} = \frac{2+\sqrt{6}-2\sqrt{6}-6}{2-3}$$

$$= \frac{-4-\sqrt{6}}{-1} = 4+\sqrt{6}$$

Jawaban: E

$$3. {}^9\log 15 = \frac{{}^3\log 15}{{}^3\log 9} = \frac{{}^3\log (3 \times 5)}{{}^3\log 3^2}$$

$$= \frac{{}^3\log 3 + {}^3\log 5}{2} = \frac{1+p}{2} = \frac{1}{2}(p+1)$$

Jawaban: E

$$4. {}^3\log x + {}^3\log (2x-3) < 3$$

Syarat pertidaksamaan

$$\Leftrightarrow {}^3\log x(2x-3) < {}^3\log 27$$

$$\Leftrightarrow 2x^2-3x < 27 \Leftrightarrow 2x^2-3x-27 < 0$$

$$\Leftrightarrow (2x-9)(x+3) < 0 \Leftrightarrow -3 < x < \frac{9}{2} \quad \dots (1)$$

Syarat numerus

$$x > 0 \quad \dots (2)$$

$$2x-3 > 0 \Leftrightarrow x > \frac{3}{2} \quad \dots (3)$$

Dari (1), (2), dan (3) diperoleh irisan $\left\{x \mid \frac{3}{2} < x < \frac{9}{2}\right\}$.

Jawaban: D

$$5. f(x) = 2x+5 \text{ dan } g(x) = \frac{x-1}{x+4}, x \neq -4$$

$$(f \circ g)(x) = f(g(x))$$

$$= f\left(\frac{x-1}{x+4}\right) = 2\left(\frac{x-1}{x+4}\right) + 5$$

$$= \frac{2x-2}{x+4} + \frac{5(x+4)}{x+4}$$

$$= \frac{2x-2+5x+20}{x+4}$$

$$= \frac{7x+18}{x+4}, x \neq -4$$

Jawaban: D

$$6. f(x) = \frac{x+4}{x-6}, x \neq 6 \text{ dan } g(x) = 2x-1$$

$$(f \circ g)(x) = f(g(x)) = f(2x-1)$$

$$= \frac{2x-1+4}{2x-1-6} = \frac{2x+3}{2x-7}, x \neq \frac{7}{2}$$

$$\text{Jika } f(x) = \frac{2x+3}{2x-7}, x \neq \frac{7}{2}, \text{ maka } f^{-1}(x) = \frac{-dx+b}{cx-a}$$

$$(f \circ g)(x) = \frac{2x+3}{2x-7}$$

$$\Rightarrow (f \circ g)^{-1} = \frac{7x+3}{2x-2}, x \neq 1$$

Jawaban: C

7. Persamaan kuadrat $x^2+ax-4=0$ akar-akarnya p dan q dengan $p^2-2pq+q^2=8a$.

$$x^2+ax-4=0$$

$$p+q = -\frac{a}{1} = -a \quad p \cdot q = \frac{-4}{1} = -4$$

$$p^2-2pq+q^2 = (p^2+q^2)-2pq$$

$$8a = ((p+q)^2-2pq)-2pq$$

$$8a = (p+q)^2-4pq$$

$$8a = (-a)^2-4(-4)$$

$$8a = a^2+16$$

$$a^2-8a+16=0$$

$$(a-4)(a-4)=0$$

$$a=4$$

Jawaban: C

8. x_1 dan x_2 adalah akar-akar persamaan kuadrat

$$x^2-2x-1=0, \text{ maka}$$

$$x_1+x_2=2 \text{ dan } x_1x_2=-1$$

$$x_1^2+x_2^2 = (x_1+x_2)^2-2x_1x_2 = 4+2=6$$

$$x_1^3+x_2^3 = (x_1+x_2)^3-3x_1x_2(x_1+x_2) = 8+6=14$$

$$\text{JAB} \Rightarrow x_1^2+x_2+x_1+x_2^2 = (x_1^2+x_2^2) + (x_1+x_2)$$

$$= 6+2=8$$

$$\text{HAB} \Rightarrow (x_1^2+x_2)(x_1+x_2^2) = x_1^3+x_1^2x_2^2+x_1x_2+x_2^3$$

$$= (x_1^3+x_2^3) + (x_1x_2)^2 + x_1x_2$$

$$= 14+1-1=14$$

$$\text{PK baru} \Rightarrow x^2 - (\text{JAB})x + \text{HAB} = 0$$

$$x^2 - 8x + 14 = 0$$

Jawaban: A

9. "Jika garis k tegak lurus bidang a , maka semua garis di bidang a tegak lurus garis k ". Pernyataan tersebut merupakan suatu implikasi yang mengandung pernyataan berkuantor (mengandung kata "semua").

Ingat: $\sim(p \Rightarrow q) \equiv p \wedge \sim q$ dan
 $\sim(\text{semua } q) \equiv \text{ada } \sim q$

Dengan demikian, negasi pernyataan di atas adalah:
 "Garis k tegak lurus bidang a , tetapi ada garis di bidang a yang tidak tegak lurus garis k ".

Jawaban: C

10. Grafik fungsi $y = (m - 2)x^2 - 2mx + m + 6$ seluruhnya berada di atas sumbu- $X \Rightarrow$ definit positif, dengan syarat $a > 0$ dan $D < 0$.

$$a > 0 \Rightarrow m - 2 > 0 \Leftrightarrow m > 2 \quad \dots (1)$$

$$D < 0 \Rightarrow b^2 - 4ac < 0$$

$$(-2m)^2 - 4(m - 2)(m + 6) < 0$$

$$4m^2 - 4m^2 - 16m + 48 < 0$$

$$-16m + 48 < 0$$

$$-16m < -48$$

$$16m > 48$$

$$m > 3 \quad \dots (2)$$

$$(1) \cap (2) = m > 3$$

Jadi, haruslah dipenuhi $m > 3$.

Jawaban: D

11. Misalkan banyak orang dewasa = a , banyak pelajar = b , dan banyak anak-anak = c .

Sehingga,

$$a + b + c = 110 \quad \dots (1)$$

$$120.000a + 70.000b + 40.000c = 10.600.000$$

$$\Leftrightarrow 12a + 7b + 4c = 1.006 \quad \dots (2)$$

$$a = 2b - 10 \Leftrightarrow a - 2b = -10 \quad \dots (3)$$

Dari persamaan (1) dan (2) diperoleh:

$$a + b + c = 110 \quad \times 4$$

$$12a + 7b + 4c = 1.006 \quad \times 1$$

$$4a + 4b + 4c = 440$$

$$12a + 7b + 4c = 1.006$$

$$-8a - 3b = -566$$

$$8a + 3b = 566 \quad \dots (4)$$

Dari persamaan (3) dan (4) diperoleh:

$$a - 2b = -10 \quad \times 8 \quad 8a - 16b = -80$$

$$8a + 3b = 566 \quad \times 1 \quad 8a + 3b = 566$$

$$-19b = -646$$

$$b = 34$$

Substitusikan $b = 34$ ke persamaan (3)

$$a = 2(34) - 10 = 68 - 10 = 58$$

Jadi, banyaknya tiket yang terjual untuk dewasa dan pelajar berturut-turut adalah 58 dan 34.

Jawaban: C

12. Misalnya: sepeda gunung = x
 sepeda balap = y

Jenis Sepeda	Harga	Jumlah	Keuntungan
Sepeda gunung	1.500.000	x	500.000
Sepeda balap	2.000.000	y	600.000
	42.000.000	25	

Model matematika:

$$x + y \leq 25,$$

$$1.500.000x + 2.000.000y \leq 42.000.000 \Leftrightarrow 3x + 4y \leq 84,$$

$$x \geq 0, y \geq 0$$

$$\text{Fungsi tujuan: maks } f(x, y) = (500.000x + 600.000y)$$

$$\text{Titik potong } x + y = 25 \text{ dan } 3x + 4y = 84$$

$$\begin{array}{rcl} x + y & = & 25 \\ 3x + 4y & = & 84 \\ \hline -y & = & -9 \\ y & = & 9 \\ \Rightarrow x & = & 25 - 9 = 16 \end{array}$$

Titik potongnya (16, 9)

Titik Potong	$f(x, y) = 500.000x + 600.000y$
(25, 0)	12.500.000
(16, 9)	$8.000.000 + 5.400.000 = 13.400.000$ (maks)
(0, 21)	12.600.000

Keuntungan maksimumnya adalah Rp13.400.000,00.

Jawaban: A

$$13. 2A - B + 3C = 2 \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} -1 & 2 \\ 5 & 6 \end{pmatrix} + 3 \begin{pmatrix} a & -1 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 2 \\ 6 & 8 \end{pmatrix} - \begin{pmatrix} -1 & 2 \\ 5 & 6 \end{pmatrix} + \begin{pmatrix} 3a & -3 \\ 6 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 5+3a & -3 \\ 7 & 11 \end{pmatrix}$$

$$\det(2A - B + 3C) = \begin{vmatrix} 5+3a & -3 \\ 7 & 11 \end{vmatrix}$$

$$\Leftrightarrow 10 = 11(5+3a) - 7(-3)$$

$$\Leftrightarrow 10 = 55 + 33a + 21$$

$$\Leftrightarrow 10 = 76 + 33a$$

$$\Leftrightarrow 10 - 76 = 33a$$

$$\Leftrightarrow 33a = 10 - 76$$

$$\Leftrightarrow 33a = -66$$

$$\Leftrightarrow a = -2$$

Jawaban: C

$$\begin{aligned}
14. \quad P \begin{pmatrix} 6 & 7 \\ 8 & 9 \end{pmatrix} &= \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \\
P &= \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 6 & 7 \\ 8 & 9 \end{pmatrix}^{-1} \\
P &= \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \frac{1}{54-56} \begin{pmatrix} 9 & -7 \\ -8 & 6 \end{pmatrix} \\
P &= \frac{1}{-2} \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 9 & -7 \\ -8 & 6 \end{pmatrix} \\
P &= \frac{1}{-2} \begin{pmatrix} 18-24 & -14+18 \\ 36-40 & -28+30 \end{pmatrix} \\
P &= \frac{1}{-2} \begin{pmatrix} -6 & 4 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}
\end{aligned}$$

Jawaban: E

15. Barisan aritmetika dengan $U_4 = 110$ dan $U_9 = 150$.

$$\begin{aligned}
U_4 &= a + 3b = 110 \\
U_9 &= a + 8b = 150 \\
\hline
-5b &= -40 \\
b &= 8 \\
U_{30} &= a + 29b = (a + 8b) + 21b \\
&= U_9 + 21b = 150 + 21(8) \\
&= 150 + 168 = 318
\end{aligned}$$

Jawaban: B

16. Dalam deret aritmetika, $S_n = \frac{n}{2}(2a + (n-1)b)$

$$\begin{aligned}
S_{12} &= \frac{12}{2} \{2(46.000) + 11(18.000)\} \\
&= 6(92.000 + 198.000) = 1.740.000
\end{aligned}$$

Jadi, jumlah keuntungan sampai bulan ke-12 adalah Rp1.740.000,00.

Jawaban: A

17. $U_5 = ar^4 = 324$

$$\begin{aligned}
4r^4 &= 324 \\
r^4 &= \frac{324}{4} = 81 \\
r &= 3
\end{aligned}$$

Dalam deret geometri, $S_n = \frac{a(r^n - 1)}{r - 1}; r > 1$

Sehingga:

$$S_8 = \frac{4(3^8 - 1)}{3 - 1} = \frac{4(6.561 - 1)}{2} = 2(6.560) = 13.120$$

Jawaban: C

18. $P(x)$ dibagi $(x + 3)$ bersisa 2, artinya $P(-3) = 2$

$$\begin{aligned}
P(-3) &= (-3)^4 + 5(-3)^3 + 9(-3)^2 + 13(-3) + a \\
2 &= 81 - 135 + 81 - 39 + a \\
2 &= -12 + a \\
a &= 12 + 2 = 14
\end{aligned}$$

$$\begin{aligned}
P(-1) &= (-1)^4 + 5(-1)^3 + 9(-1)^2 + 13(-1) + 14 \\
&= 1 - 5 + 9 - 13 + 14 = 6
\end{aligned}$$

Jawaban: E

19. $\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} = \frac{0}{0}$ (bentuk tak tentu)

$$\begin{aligned}
\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} &= \lim_{x \rightarrow 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2} \\
&= \lim_{x \rightarrow 4} (\sqrt{x}+2) = \sqrt{4} + 2 = 2 + 2 = 4
\end{aligned}$$

Jawaban: B

$$\begin{aligned}
20. \quad \lim_{x \rightarrow \infty} \left\{ (3x-2) - \sqrt{9x^2 - 2x + 5} \right\} \\
&= \frac{2(3)(-2) - (-2)}{2\sqrt{9}} \\
&= -\frac{5}{3}
\end{aligned}$$

$$\begin{aligned}
&\lim_{x \rightarrow \infty} \left\{ (bx+c) - \sqrt{ax^2 + px + q} \right\} \\
&= \frac{2bc-p}{2\sqrt{a}}
\end{aligned}$$

Jawaban: E

21. $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 1$

$$f'(x) = x^2 + 2x - 15$$

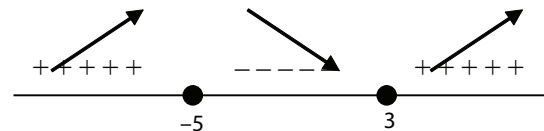
$$f''(x) = 2x + 2$$

Nilai stasioner $f'(x) = 0$

$$x^2 + 2x - 15 = 0$$

$$(x+5)(x-3) = 0$$

$$x = -5 \text{ atau } x = 3$$



Berdasarkan garis bilangan di atas dapat diperoleh fungsi f mencapai nilai maksimum relatif untuk nilai $x = -5$.

Jawaban: B

$$22. \int 3x\sqrt{3x^2+1} dx =$$

$$\text{Misal: } u = 3x^2 + 1 \Rightarrow du = 6x dx \Rightarrow \frac{du}{2} = 3x dx$$

$$\begin{aligned}
\int 3x\sqrt{3x^2+1} dx &= \int \sqrt{u} \frac{du}{2} = \frac{1}{2} \int u^{\frac{1}{2}} du \\
&= \frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} + C = \frac{1}{3} u\sqrt{u} + C \\
&= \frac{1}{3} (3x^2+1)\sqrt{3x^2+1} + C
\end{aligned}$$

Jawaban: C

23. $\int_0^1 5x(1-x)^6 dx$

turunkan	integrasikan	
$5x$	$(1-x)^6$	
5	$\frac{1}{7} \cdot \frac{1}{-1} \cdot (1-x)^7$	+
0	$\frac{1}{56} (1-x)^8$	-

$$\begin{aligned} \int_0^1 5x(1-x)^6 dx &= 5x \cdot -\frac{1}{7}(1-x)^7 - 5 \cdot \frac{1}{56}(1-x)^8 \Big|_0^1 \\ &= \left[-\frac{5(1)}{7}(1-1)^7 - \frac{5}{56}(1-1)^8 \right] \\ &\quad - \left[-\frac{5(0)}{7}(1-0)^7 - \frac{5}{56}(1-0)^8 \right] \\ &= [0-0] - \left[0 - \frac{5}{56} \right] = \frac{5}{56} \end{aligned}$$

Jawaban: B

24. Keuntungan = harga jual - biaya produksi

$$\begin{aligned} K(x) &= 40x - (4x^2 - 8x + 24)x = 40x - 4x^3 + 8x^2 - 24x \\ &= -4x^3 + 8x^2 + 16x \end{aligned}$$

Keuntungan maksimal $K(x) = 0$

$$\begin{aligned} K'(x) &= -12x^2 + 16x + 16 = 0 \\ -3x^2 + 4x + 4 &= 0 \\ (-3x-2)(x-2) &= 0 \end{aligned}$$

$$x = \frac{2}{3} \text{ atau } x = 2$$

$$\begin{aligned} \text{Keuntungan} &= -4(2)^3 + 8(2)^2 + 16(2) \\ &= -4(8) + 8(4) + 32 = 32 \end{aligned}$$

Jadi, keuntungan maksimum yang diperoleh perusahaan adalah Rp32.000,00.

Jawaban: B

25. $\alpha - \beta = \frac{\pi}{3}$ dan $\sin \alpha \sin \beta = \frac{1}{4}$ (α, β sudut lancip)

$$\cos(\alpha - \beta) = \cos \frac{\pi}{3}$$

$$\cos \alpha \cos \beta + \sin \alpha \sin \beta = \frac{1}{2}$$

$$\cos \alpha \cos \beta + \frac{1}{4} = \frac{1}{2}$$

$$\cos \alpha \cos \beta = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = \frac{1}{4} - \frac{1}{4} = 0$$

Jawaban: E

$$\begin{aligned} 26. \frac{\sin 150^\circ + \sin 120^\circ}{\cos 210^\circ - \cos 300^\circ} &= \frac{\sin 30^\circ + \sin 60^\circ}{-\cos 30^\circ - \cos 60^\circ} = \frac{\frac{1}{2} + \frac{1}{2}\sqrt{3}}{-\frac{1}{2}\sqrt{3} - \frac{1}{2}} \\ &= \frac{\frac{1+\sqrt{3}}{2}}{\frac{-\sqrt{3}-1}{2}} = \frac{1+\sqrt{3}}{-\sqrt{3}-1} = -\frac{1+\sqrt{3}}{1+\sqrt{3}} = -1 \end{aligned}$$

Jawaban: B

27. $\cos 2x + \cos x = 0; 0^\circ \leq x \leq 180^\circ$

$$(2\cos^2 x - 1) + \cos x = 0 \quad \text{Ingat: } \cos 2x = 2\cos^2 x - 1$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

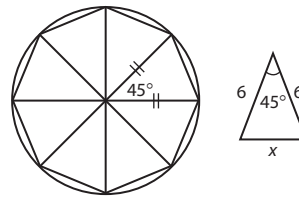
$$\cos x = \frac{1}{2} \text{ atau } \cos x = -1$$

$$x = 60^\circ \text{ atau } x = 180^\circ$$

Jadi, himpunan penyelesaiannya $\{60^\circ, 180^\circ\}$.

Jawaban: E

28. Lingkaran berjari-jari 6 cm dibuat segi-8 beraturan.



Dengan menggunakan aturan kosinus

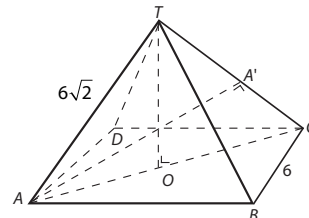
$$\begin{aligned} x^2 &= 6^2 + 6^2 - 2(6)(6)\cos 45^\circ \\ &= 36 + 36 - 72\left(\frac{1}{2}\sqrt{2}\right) = 72 - 36\sqrt{2} \\ x &= \sqrt{72 - 36\sqrt{2}} = 6\sqrt{2 - \sqrt{2}} \end{aligned}$$

Jadi, panjang sisi segi-8 beraturan tersebut adalah

$$6\sqrt{2 - \sqrt{2}} \text{ cm.}$$

Jawaban: A

- 29.



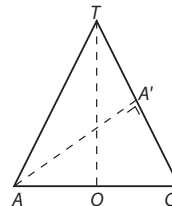
$AC = \text{diagonal sisi alas} = 6\sqrt{2} \text{ cm}$

$$OC = \frac{1}{2}AC = \frac{1}{2}(6\sqrt{2}) = 3\sqrt{2}$$

$$TO = \sqrt{TC^2 - OC^2}$$

$$= \sqrt{(6\sqrt{2})^2 - (3\sqrt{2})^2} = \sqrt{72 - 18} = \sqrt{54} = 3\sqrt{6}$$

Perhatikan segitiga ATC yang merupakan segitiga samasisi.



$$\text{Luas} = \frac{1}{2} \times AC \times TO = \frac{1}{2} \times TC \times AA'$$

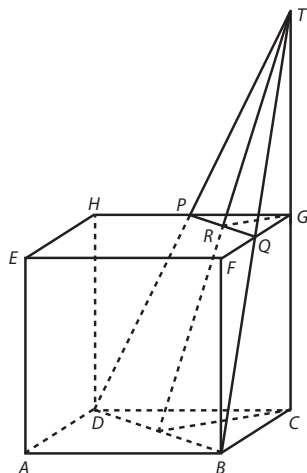
$$6\sqrt{2} \times 3\sqrt{6} = 6\sqrt{2} \times AA'$$

$$AA' = 3\sqrt{6}$$

Jadi, jarak A ke garis TC adalah $3\sqrt{6}$.

Jawaban: D

30. Perhatikan gambar berikut.



Perhatikan $\triangle TBC$.

$$BC = 4 \quad TC = 4 + 4 = 8$$

$\triangle TQG$ kongruen dengan $\triangle TBC$, sehingga

$$\frac{GQ}{BC} = \frac{GT}{TC} \Leftrightarrow \frac{GQ}{4} = \frac{4}{8} \Leftrightarrow GQ = 2$$

Karena $\triangle TCD$ sama dan sebangun dengan $\triangle TBC$, maka $PG = GQ = 2$.

Selanjutnya perhatikan bahwa $\triangle PQG$ siku-siku di G .

$$PQ = \sqrt{PG^2 + GQ^2} = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

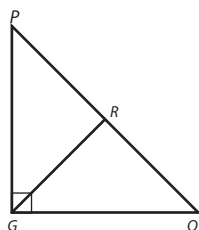
Dari luas $\triangle PQG$,

$$\frac{1}{2} \cdot GQ \cdot PG = \frac{1}{2} \cdot QP \cdot GR$$

$$2 \cdot 2 = 2\sqrt{2} \cdot GR$$

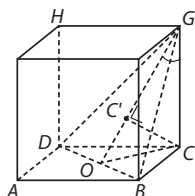
$$GR = \sqrt{2}$$

$$\tan \alpha = \frac{GR}{GT} = \frac{\sqrt{2}}{4}$$



Jawaban: D

31. Kubus $ABCD.EFGH$ dengan rusuk = 10 cm



Perhatikan $\triangle GOC$

$$CO = \frac{1}{2} \text{ diagonal bidang} = \frac{1}{2}(10\sqrt{2}) = 5\sqrt{2}$$

$$\begin{aligned} GO &= \sqrt{GC^2 + OC^2} \\ &= \sqrt{10^2 + (5\sqrt{2})^2} = \sqrt{100 + 50} = \sqrt{150} = 5\sqrt{6} \end{aligned}$$

$$\begin{aligned} GC' &= \frac{2}{3}GO \quad (C' \text{ titik berat } \triangle GBD) \\ &= \frac{2}{3}(5\sqrt{6}) = \frac{10}{3}\sqrt{6} \end{aligned}$$

$$\cos \angle (GC, BDG) = \frac{GC'}{GC} = \frac{\frac{10}{3}\sqrt{6}}{10} = \frac{1}{3}\sqrt{6}$$

Jawaban: A

$$\begin{aligned} 32. \cos \angle (\vec{a} \cdot \vec{b}) &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{6 + 6 - 12}{\sqrt{2^2 + (-3)^2 + 3^2} \cdot \sqrt{3^2 + (-2)^2 + (-4)^2}} = 0 \\ \angle (\vec{a} \cdot \vec{b}) &= 90^\circ \end{aligned}$$

Jawaban: C

33. Ingat: Jika lingkaran berpusat (x_1, y_1) menyinggung garis

$$Ax + By + C = 0, \text{ maka } R = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Sehingga:

$$R = \frac{|3(2) + (-4)(-3) + 2|}{\sqrt{3^2 + (-4)^2}} = \frac{|6 + 12 + 2|}{\sqrt{25}} = \frac{20}{5} = 4$$

Persamaan lingkaran dengan pusat $(2, -3)$ dan jari-jari 4 adalah:

$$(x - 2)^2 + (y - (-3))^2 = 4^2 \Leftrightarrow (x - 2)^2 + (y + 3)^2 = 16$$

Jawaban: A

34. $2x - y + 7 = 0 \Rightarrow y = 2x + 7$, gradiennya = 2 = gradien garis singgung

$$x^2 + y^2 - 6x - 2y + 5 = 0$$

$$(x - 3)^2 - 9 + (y - 1)^2 - 1 + 5 = 0$$

$$(x - 3)^2 + (y - 1)^2 = 5$$

Garis singgung lingkaran $(x - a)^2 + (y - b)^2 = R^2$ dengan gradien m adalah:

$$y - b = m(x - a) \pm R\sqrt{1 + m^2}$$

Dengan demikian diperoleh:

$$y - 1 = 2(x - 3) \pm \sqrt{5} \cdot \sqrt{1 + 2^2}$$

$$y - 1 = 2x - 6 \pm 5$$

$$2x - y - 5 + 5 = 0 \Leftrightarrow 2x - y = 0$$

$$2x - y - 5 - 5 = 0 \Leftrightarrow 2x - y - 10 = 0$$

Jawaban: A

$$\begin{aligned} 35. \begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \Leftrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \\ \Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix} \Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-\frac{1}{4}} \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \\ \Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \end{aligned}$$

Diperoleh $x = 2x'$ dan $y = -2y'$

Kita substitusikan ke persamaan kurva,

$$-2y' = \sin 2x'$$

$$\Leftrightarrow y = -\frac{1}{2} \sin(2x) = -\frac{1}{2} (2 \sin x \cos x)$$

$$y = -\sin x \cos x$$

Jawaban: D

36.

Ukuran	f
1-5	3
6-10	17
11-15	18
16-20	22
21-25	25
26-30	21
31-35	4

← Kelas modus

Frekuensi terbanyak adalah 25, sehingga kelas modus terletak di 21-25.

$$T_b = 20,5$$

$$s_1 = 25 - 22 = 3$$

$$s_2 = 25 - 21 = 4$$

Panjang kelas, $C = 5$

$$Mo = T_b + \frac{s_1}{s_1 + s_2} \cdot C$$

$$Mo = 20,5 + \left(\frac{3}{3+4}\right)5 = 20,5 + \frac{3}{7} \cdot 5$$

Jawaban: C

37.

Berat Badan (kg)	f	F _{kum}
36-45	5	5
46-55	10	15
56-65	12	27
66-75	7	34
76-85	6	40

$$\text{Kelas } Q_3 = \frac{3}{4} (40) = 30 \text{ terletak pada interval } 66-75$$

$$T_b = 65,5; f_s = 27; f(Q_3) = 7; C = 10$$

$$Q_3 = T_b + \frac{\left(\frac{3}{4}n - f_s\right)}{f(Q_3)} \cdot C = 65,5 + \frac{30 - 27}{7} \cdot 10 = 65,5 + \frac{30}{7}$$

Jawaban: A

38. Dengan menggunakan aturan pengisian tempat.

Ribuan	Ratusan	Puluhan	Satuan
6	5	4	3

Banyak susunan bilangannya adalah $6 \times 5 \times 4 \times 3 = 360$.

Jawaban: E

39. Banyaknya tim yang mungkin dibentuk

$$C_{(9,5)} = \frac{9!}{5!(9-5)!} = \frac{9 \times 8 \times 7 \times 6 \times 5!}{5! \times 4 \times 3 \times 2 \times 1} = 9 \times 7 \times 2 = 126$$

Jawaban: A

$$40. \text{ Kotak I, } n(S) = C_{(5,2)} = \frac{5!}{2!(5-2)!} = \frac{5 \times 4 \times 3!}{2 \times 1 \times 3!} = 10$$

A = terambil bola merah

$$n(A) = C_{(3,2)} = \frac{3!}{2!(3-2)!} = \frac{3 \times 2!}{2! \times 1} = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{10}$$

$$\text{Kotak II, } n(S) = C_{(8,2)} = \frac{8!}{2!(8-2)!} = \frac{8 \times 7 \times 6!}{2 \times 1 \times 6!} = 28$$

B = terambil 2 bola biru

$$n(B) = C_{(5,2)} = \frac{5!}{2!(5-2)!} = \frac{5 \times 4 \times 3!}{2 \times 1 \times 3!} = 10$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{10}{28}$$

$$P(A \cap B) = P(A) \times P(B) = \frac{3}{10} \times \frac{10}{28} = \frac{3}{28}$$

Jawaban: B

PEMBAHASAN DAN KUNCI JAWABAN PEMAHAMAN PAKET 3

MATEMATIKA IPA

$$1. \left(\frac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\right)^{-1} (a^{\frac{2}{3}} \cdot b^{\frac{1}{2}})^{\frac{1}{2}} = \left(\frac{b^{\frac{1}{2}}}{a^{\frac{2}{3}}}\right) (a^{\frac{2}{3}} b^{\frac{1}{2}})^{\frac{1}{2}} \times \frac{a^{\frac{1}{3}}}{b^{\frac{1}{2}}} \\ = b^{\frac{1}{2} \times \frac{1}{2}} \times \frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}} = \frac{ba^{\frac{1}{2}}}{b^{\frac{1}{2}}} \\ = b^{1-\frac{1}{2}} a^{\frac{1}{2}} = a^{\frac{1}{2}} b^{\frac{1}{2}}$$

Jawaban: E

$$2. \frac{x^{-1} - y^{-1}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}} = \frac{\frac{1}{x} - \frac{1}{y}}{\sqrt{x} + \sqrt{y}} = \frac{\frac{y-x}{xy}}{\sqrt{x} + \sqrt{y}} = \frac{y-x}{xy(\sqrt{x} + \sqrt{y})}$$

Kita rasionalkan menjadi

$$\frac{y-x}{xy(\sqrt{x} + \sqrt{y})} \cdot \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \frac{-(x-y)(\sqrt{x} - \sqrt{y})}{xy(x-y)} \\ = \frac{\sqrt{y} - \sqrt{x}}{xy}$$

Jawaban: B

$$3. \left. \begin{array}{l} {}^5\log 3 = a \rightarrow {}^3\log 5 = \frac{1}{a} \\ {}^3\log 4 = b \rightarrow {}^4\log 3 = \frac{1}{b} \end{array} \right\} {}^4\log 3 \cdot {}^3\log 5 = {}^4\log 5 \\ = \frac{1}{b} \cdot \frac{1}{a} = \frac{1}{ab} \\ {}^4\log 15 = {}^4\log(3 \cdot 5) = {}^4\log 3 + {}^4\log 5 = \frac{1}{b} + \frac{1}{ab} \\ = \frac{a+1}{ab} = \frac{1+a}{ab}$$

Jawaban: A

$$4. \begin{array}{l} 5^{2x} - 6 \cdot 5^{x+1} + 125 > 0, x \in R \\ (5^x)^2 - 6 \cdot 5^x \cdot 5 + 125 > 0 \\ (5^x)^2 - 30(5^x) + 125 > 0 \\ (5^x - 5)(5^x - 25) > 0 \end{array}$$

Pembuat nol fungsi $x = 1, x = 2$

$$\begin{array}{c} + \quad - \quad + \\ \hline 1 \quad 2 \end{array}$$

$x < 1$ atau $x > 2$

Jawaban: D

$$5. f(x) = 3x - 1 \text{ dan } g(x) = 2x^2 - 3 \\ (g \circ f)(x) = g(f(x)) = g(3x - 1) \\ = 2(3x - 1)^2 - 3 = 2(9x^2 - 6x + 1) - 3 \\ = 18x^2 - 12x + 2 - 3 = 18x^2 - 12x - 1$$

Jawaban: E

$$6. f(x) = \frac{2x+1}{x-3}, x \neq 3$$

$$\text{Jika } f(x) = \frac{ax+b}{cx+d}, \text{ maka } f^{-1}(x) = \frac{-dx+b}{cx-a}$$

$$f(x) = \frac{2x+1}{x-3}, x \neq 3$$

$$\Rightarrow f^{-1}(x) = \frac{3x+1}{x-2}, x \neq 2$$

$$f^{-1}(x-2) = \frac{3(x-2)+1}{(x-2)-2}$$

$$f^{-1}(x-2) = \frac{3x-6+1}{x-2-2}$$

$$f^{-1}(x-2) = \frac{3x-5}{x-4}, x \neq 4$$

Jawaban: D

$$7. \text{ Persamaan } 2x^2 + mx + 16 = 0 \text{ dengan akar-akar positif } \alpha \text{ dan } \beta, \text{ dengan } \alpha = 2\beta$$

Ingat: Jumlah dan hasil kali akar-akar persamaan kuadrat

$$x_1 + x_2 = -\frac{b}{a} \text{ dan } x_1 \cdot x_2 = \frac{c}{a}$$

Dari persamaan di atas,

$$\bullet \alpha + \beta = -\frac{m}{2}$$

$$2\beta + \beta = -\frac{m}{2}$$

$$3\beta = -\frac{m}{2} \dots (1)$$

$$\bullet \alpha \cdot \beta = \frac{16}{2}$$

$$2\beta \cdot \beta = 8$$

$$\beta^2 = 4$$

$$\beta = 2 \text{ (karena diketahui } \beta \text{ positif)}$$

$$\text{diperoleh } 3\beta = -\frac{m}{2}$$

$$3 \cdot 2 = -\frac{m}{2}$$

$$m = -12$$

Jawaban: A

$$8. \text{ Persamaan } x^2 + (m-2)x + 2m - 4 = 0 \text{ mempunyai akar-akar real, berarti } D \geq 0.$$

$$D \geq 0 \Leftrightarrow b^2 - 4ac \geq 0$$

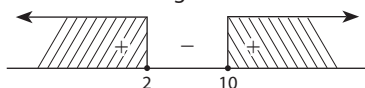
$$(m-2)^2 - 4(1)(2m-4) \geq 0$$

$$m^2 - 4m + 4 - 8m + 16 \geq 0$$

$$m^2 - 12m + 20 \geq 0$$

$$(m-2)(m-10) \geq 0$$

Pembuat nol fungsi: $m = 2$ dan $m = 10$



Jadi, batas yang memenuhi adalah $m \leq 2$ atau $m \geq 10$.

Jawaban: A

9. Premis 1 : Jika hari ini hujan deras, maka Bona tidak keluar rumah

Premis 2 : Bona keluar rumah

Kedua premis di atas dapat dimisalkan sebagai berikut.

$$P_1 : p \Rightarrow q$$

$$P_2 : \sim q$$

$$\therefore \sim p \quad (\text{Modus Tollens})$$

Jadi, kesimpulan yang sah adalah "hari ini hujan tidak deras."

Jawaban: B

10. Grafik terbuka ke bawah $\Rightarrow a < 0$

Memotong sumbu-X di dua titik $\Rightarrow D > 0$

Kedua akar positif $\Rightarrow -\frac{b}{a} > 0$ dan $\frac{c}{a} > 0$

Pernyataan yang benar adalah (1), (2), dan (3).

Jawaban: A

11. Misalkan: Umur Deksa = D

$$\text{Umur Elisa} = E$$

$$\text{Umur Firda} = F$$

Kita ubah D dan F dalam bentuk E .

$$D = E + 4$$

$$E = F + 3 \Rightarrow F = E - 3$$

$$D + E + F = 58$$

$$(E + 4) + E + (E - 3) = 58$$

$$3E + 1 = 58$$

$$3E = 57$$

$$E = 19$$

$$D + F = 58 - E = 58 - 19 = 39$$

Jadi, jumlah umur Deksa dan Firda adalah 39 tahun.

Jawaban: D

12. Misalkan: banyaknya permen $A = x$
banyaknya permen $B = y$

Jenis Permen	Daya Tampung	Harga Beli	Keuntungan
A	x	1.000	100
B	y	1.500	200
	250	300.000	

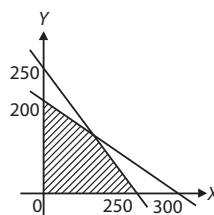
Model matematika: $x + y \leq 250$,

$$1.000x + 1.500y \leq 300.000 \Leftrightarrow 2x + 3y \leq 600,$$

$$x \geq 0, y \geq 0$$

Fungsi tujuan: maksimum $f(x, y) = 100x + 200y$

Sketsa gambar:



Titik potong $x + y = 250$ dan $2x + 3y = 600$

$$x + y = 250 \Rightarrow 2x + 2y = 500$$

$$2x + 3y = 600$$

$$-y = -100$$

$$y = 100 \Rightarrow x + 100 = 250$$

$$x = 250 - 100$$

$$= 150$$

Titik potongnya (150, 100)

Titik Pojok	$f(x, y) = 100x + 200y$
(250, 0)	25.000
(150, 100)	15.000 + 20.000 = 35.000
(0, 200)	40.000 (maksimum)

Jadi, agar keuntungan maksimum ia harus membeli 200 bungkus permen B saja.

Jawaban: E

$$13. \begin{pmatrix} 4 & x-2 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} -6 & 8 \\ -11 & -6 \end{pmatrix} = 2 \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} 0 & 3 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 4-6 & x-2+8 \\ 3-11 & 2-6 \end{pmatrix} = 2 \begin{pmatrix} 0-1 & 9+1 \\ 0-4 & -6+4 \end{pmatrix}$$

$$\begin{pmatrix} -2 & x+6 \\ -8 & -4 \end{pmatrix} = 2 \begin{pmatrix} -1 & 10 \\ -4 & -2 \end{pmatrix}$$

Diperoleh:

$$x + 6 = 2(10)$$

$$x + 6 = 20$$

$$x = 14$$

Jawaban: B

$$14. AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} -6 & -5 \\ 5 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -6+10 & -5+8 \\ -18+20 & -15+16 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$$

Ingat: Jika $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, maka $A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$(AB)^{-1} = \frac{1}{4-6} \begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{pmatrix}$$

Jawaban: E

15. Deret aritmetika dengan $S_n = 2n^2 + 4n$.

$$U_n = S_n - S_{n-1}$$

$$\begin{aligned} U_9 &= S_9 - S_8 \\ &= (2(9)^2 + 4(9)) - (2(8)^2 + 4(8)) \\ &= 198 - 160 = 38 \end{aligned}$$

Jawaban: C

16. Penjualan bulan Januari, $U_1 = a = 120$ kg

Penjualan bulan Februari, $U_2 = 130$ kg

Maret dan seterusnya selama 10 bulan bertambah 10 kg

Soal di atas merupakan soal deret aritmetika dengan $a = 120$ kg dan $b = 10$ kg

$$S_n = \frac{n}{2}(2a + (n-1)b)$$

$$\begin{aligned} S_{10} &= \frac{10}{2}(2 \cdot 120 + 9(10)) \\ &= 5(240 + 90) = 5(330) = 1.650 \end{aligned}$$

Jadi, jumlah daging yang terjual selama 10 bulan adalah 1.650 kg.

Jawaban: D

17. Tahun 2010 = 1 juta

Tahun 2000 = 2×1 juta = 2 juta

Tahun 1990 = 2×2 juta = 4 juta

Tahun 1980 = 2×4 juta = 8 juta

Tahun 1970 = 2×8 juta = 16 juta

Jawaban: C

18. $f(x) = P(x) \cdot H(x) + S$

$$f(x) = (x^2 - x - 6) \cdot (x + h) + 5x - 2$$

$$f(x) = (x^2 - 2x - 3) \cdot (x + k) + 3x + 4$$

$$\text{Faktor dari: } (x^2 - x - 6) = 0$$

$$(x + 2)(x - 3) = 0$$

$$x_1 = -2, x_2 = 3$$

$$S = f(-2) = 5(-2) - 2 = -12$$

$$S = f(3) = 5(3) - 2 = 13$$

$$f(x) = (x^2 - 2x - 3) \cdot (x + k) + 3x + 4$$

$$f(-2) = (4 + 4 - 3) \cdot (-2 + k) + -6 + 4$$

$$-12 = 5(-2 + k) - 2$$

$$-10 = -10 + 5k$$

$$k = 0$$

$$f(x) = (x^2 - 2x - 3) \cdot (x + 0) + 3x + 4$$

$$f(x) = x^3 - 2x^2 - 3x + 3x + 4$$

$$f(x) = x^3 - 2x^2 + 4$$

Jawaban: D

$$\begin{aligned} 19. \lim_{x \rightarrow 1} \frac{1-x}{2-\sqrt{x+3}} &= \lim_{x \rightarrow 1} \frac{1-x}{2-\sqrt{x+3}} \cdot \frac{2+\sqrt{x+3}}{2+\sqrt{x+3}} \\ &= \lim_{x \rightarrow 1} \frac{(1-x)(2+\sqrt{x+3})}{2^2 - (\sqrt{x+3})^2} \\ &= \lim_{x \rightarrow 1} \frac{(1-x)(2+\sqrt{x+3})}{4 - (x+3)} \\ &= \lim_{x \rightarrow 1} \frac{(\cancel{x+1})(2+\sqrt{x+3})}{(\cancel{x+1})} \\ &= \lim_{x \rightarrow 1} (2 + \sqrt{x+3}) = 2 + \sqrt{1+3} = 2 + 2 = 4 \end{aligned}$$

Jawaban: B

$$\begin{aligned} 20. \lim_{x \rightarrow \infty} (\sqrt{x^2 - 5x} - x - 2) &= \lim_{x \rightarrow \infty} (\sqrt{x^2 - 5x} - x) - 2 \\ &= \frac{-5-0}{2\sqrt{1}} - 2 = \frac{-5}{2} - 2 \\ &= \frac{-5}{2} - \frac{4}{2} = -\frac{9}{2} \end{aligned}$$

Jawaban: A

21. $f(x) = x^3 + 3x^2 + 5$

$$f'(x) = 3x^2 + 6x$$

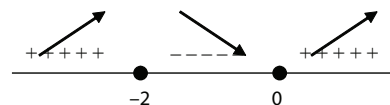
$$f''(x) = 6x + 6$$

Nilai stasioner jika $f'(x) = 0$

$$3x^2 + 6x = 0$$

$$3x(x + 2) = 0$$

$$x = -2 \text{ atau } x = 0$$



Berdasarkan garis bilangan di atas dapat diperoleh fungsi f mencapai nilai minimum relatif untuk $x = 0$.

Jawaban: C

$$22. \int \frac{3x-1}{(3x^2-2x+7)^7} dx$$

$$\text{Misal: } u = 3x^2 - 2x + 7$$

$$\text{maka } du = (6x - 2)dx \Leftrightarrow \frac{du}{2} = (3x - 1)dx$$

$$\begin{aligned} \int \frac{3x-1}{(3x^2-2x+7)^7} dx &= \int \frac{1}{u^7} \cdot \frac{du}{2} = \frac{1}{2} \int u^{-7} du = \frac{1}{-12} u^{-6} + C \\ &= \frac{-1}{12u^6} + C = \frac{-1}{12(3x^2-2x+7)^6} + C \end{aligned}$$

Jawaban: D

$$\begin{aligned}
 23. \int_2^4 (-x^2 + 6x - 8) dx &= -\frac{1}{3}x^3 + 3x^2 - 8x \Big|_2^4 \\
 &= \left(-\frac{1}{3}(4)^3 + 3(4)^2 - 8(4)\right) \\
 &\quad - \left(-\frac{1}{3}(2)^3 + 3(2)^2 - 8(2)\right) \\
 &= \left(-\frac{64}{3} + 16\right) - \left(-\frac{8}{3} - 4\right) \\
 &= -\frac{64}{3} + \frac{8}{3} + 20 \\
 &= -\frac{56}{3} + 20 = \frac{-56 + 60}{3} = \frac{4}{3}
 \end{aligned}$$

Jawaban: E

24. Keuntungan = harga jual – biaya produksi

$$\begin{aligned}
 K(x) &= 50x - (5x^2 - 10x + 30) \\
 &= 50x - 5x^2 + 10x - 30 \\
 &= -5x^2 + 10x + 20
 \end{aligned}$$

$$\begin{aligned}
 \text{Keuntungan maksimum, } K'(x) &= 0 \\
 -15x^2 + 20x + 20 &= 0 \\
 -3x^2 + 4x + 4 &= 0 \\
 (-3x - 2)(x - 2) &= 0 \\
 x &= -\frac{2}{3} \text{ atau } x = 2
 \end{aligned}$$

$$\begin{aligned}
 \text{Untuk } x = 2, K(2) &= -5(2)^2 + 10(2) + 20 \\
 &= -40 + 40 + 20 = 20
 \end{aligned}$$

Jadi, keuntungan maksimumnya adalah Rp40.000,00.

Jawaban: D

25. Pada segitiga ABC berlaku

$$A + B + C = 180^\circ.$$

Siku-siku di C berarti $C = 90^\circ$,

sehingga $A + B = 90^\circ$

$$\cos(A + B) = \cos 90^\circ \Leftrightarrow \cos A \cos B - \sin A \sin B = 0$$

$$\Leftrightarrow \cos A \cos B = \sin A \sin B \Leftrightarrow \cos A \cos B = \frac{2}{5}$$

Selanjutnya,

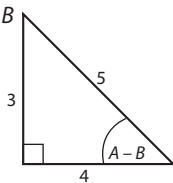
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{2}{5} + \frac{2}{5} = \frac{4}{5}$$

$$\sin(A - B) = \frac{3}{5}$$

$$\text{sehingga } 5a = \frac{3}{5} \Leftrightarrow a = \frac{3}{25}$$

$$\begin{aligned}
 \cos(a \pm b) \\
 &= \cos a \cos b \mp \\
 &\quad \sin a \sin b
 \end{aligned}$$



Jawaban: D

$$\begin{aligned}
 26. \frac{\cos 140^\circ - \cos 100^\circ}{\sin 140^\circ - \sin 100^\circ} &= \frac{-2 \sin \frac{1}{2}(140^\circ + 100^\circ) \sin \frac{1}{2}(140^\circ - 100^\circ)}{2 \cos \frac{1}{2}(140^\circ + 100^\circ) \sin \frac{1}{2}(140^\circ - 100^\circ)} \\
 &= \frac{-2 \sin 120^\circ}{2 \cos 120^\circ} = \frac{-\sin 60^\circ}{-\cos 60^\circ} = \frac{-\frac{1}{2}\sqrt{3}}{-\frac{1}{2}} = \sqrt{3}
 \end{aligned}$$

Jawaban: E

$$27. \cos 2x - 2\cos x = -1; 0 \leq x \leq 2\pi$$

$$(2\cos^2 x - 1) - 2\cos x + 1 = 0$$

$$2\cos^2 x - 2\cos x = 0$$

$$2\cos x (\cos x - 1) = 0$$

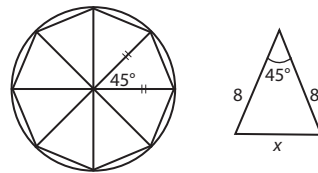
$$\cos x = 0 \text{ atau } \cos x = 1$$

$$x = \frac{\pi}{2}, \frac{3}{2}\pi \quad x = 0, 2\pi$$

Himpunan penyelesaiannya adalah $\left\{0, \frac{1}{2}\pi, \frac{3}{2}\pi, 2\pi\right\}$.

Jawaban: A

28. Lingkaran berjari-jari 8 cm dibuat segi-8 beraturan.



Dengan menggunakan aturan kosinus:

$$x^2 = 8^2 + 8^2 - 2 \cdot 8 \cdot 8 \cdot \cos 45^\circ$$

$$x^2 = 64 + 64 - 128 \cdot \frac{1}{2}\sqrt{2}$$

$$x^2 = 128 - 64\sqrt{2}$$

$$x = \sqrt{128 - 64\sqrt{2}}$$

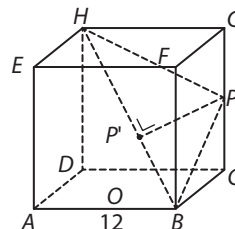
Jadi, panjang sisi segi-8 beraturan tersebut adalah

$$\sqrt{128 - 64\sqrt{2}} \text{ cm.}$$

Jawaban: B

29. Kubus ABCD EFGH dengan rusuk 12 cm

P titik tengah CG.



$$HB = \text{diagonal ruang} = 12\sqrt{3} \text{ cm}$$

Perhatikan $\triangle BCP$

$$BP = \sqrt{BC^2 + CP^2}$$

$$= \sqrt{12^2 + 6^2} = \sqrt{144 + 36}$$

$$= \sqrt{180} = 6\sqrt{5}$$

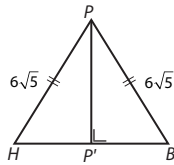
Perhatikan $\triangle HGP$

$$HP = \sqrt{HG^2 + GP^2}$$

$$= \sqrt{12^2 + 6^2} = \sqrt{144 + 36}$$

$$s = \sqrt{180} = 6\sqrt{5}$$

Perhatikan $\triangle HPB$



$$HP' = \frac{1}{2} HB = \frac{1}{2} (12\sqrt{3}) \text{ cm} = 6\sqrt{3} \text{ cm}$$

$$\begin{aligned} PP' &= \sqrt{HP^2 - (HP')^2} \\ &= \sqrt{(6\sqrt{5})^2 - (6\sqrt{3})^2} = \sqrt{180 - 108} = \sqrt{72} = 6\sqrt{2} \end{aligned}$$

Jadi, jarak P ke garis HB adalah $6\sqrt{2}$ cm.

Jawaban: D

30. Limas beraturan $T.ABCD$ dengan rusuk alas = 2 cm dan rusuk tegak = $\sqrt{3}$ cm.

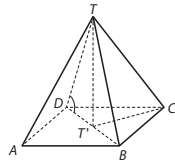
$$\tan \angle(TD, ABCD) = \frac{TT'}{T'D}$$

Perhatikan $\triangle TT'C$

$$T'C = \frac{1}{2} \text{ diagonal sisi alas} = \frac{1}{2} (2\sqrt{2}) = \sqrt{2}$$

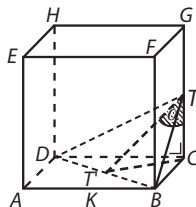
$$TT' = \sqrt{TC^2 - (T'C)^2} = \sqrt{(\sqrt{3})^2 - (\sqrt{2})^2} = \sqrt{3-2} = 1$$

$$\tan \angle(TD, ABCD) = \frac{TT'}{T'D} = \frac{TT'}{T'C} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$$



Jawaban: B

- 31.



$$TC = \frac{1}{2} GC = \frac{1}{2} (4) = 2 \text{ cm}; AC = \text{diagonal sisi} = 4\sqrt{2} \text{ cm}$$

$$T'C = \frac{1}{2} AC = \frac{1}{2} (4\sqrt{2}) = 2\sqrt{2} \text{ cm}$$

$$\begin{aligned} TT' &= \sqrt{(T'C)^2 + TC^2} \\ &= \sqrt{(2\sqrt{2})^2 + 2^2} = \sqrt{8+4} = \sqrt{12} = 2\sqrt{3} \end{aligned}$$

$$\sin \alpha = \frac{T'C}{TT'} = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

Jawaban: E

32. Jika vektor $\vec{a}$ dan $\vec{b}$ saling tegak lurus, maka $\vec{a} \cdot \vec{b} = 0$

Diketahui:

$$\vec{a} = \begin{pmatrix} -3 \\ 1 \\ -2 \end{pmatrix} \text{ dan } \vec{b} = \begin{pmatrix} -2 \\ 4 \\ x \end{pmatrix}$$

$$\vec{a} \cdot \vec{b} = 0$$

$$-3(-2) + 1(4) + (-2)(x) = 0$$

$$6 + 4 - 2x = 0$$

$$-2x = -10$$

$$x = 5$$

Jawaban: E

33. Pusat $(-1, 5)$ dan menyinggung garis $x = 1$, maka $r = 2$.

Persamaan lingkarannya:

$$(x+1)^2 + (y-5)^2 = 2^2$$

$$x^2 + 2x + 1 + y^2 - 10y + 25 = 4$$

$$x^2 + y^2 + 2x - 10y + 22 = 0$$

Jawaban: E

34. Misalkan persamaan garis singgungnya:

$$y = mx + b \quad \dots(1)$$

$$\text{Melalui titik } (1, 6) \Rightarrow 6 = m + b \Leftrightarrow b = 6 - m \quad \dots(2)$$

Substitusikan (1) ke persamaan lingkaran:

$$x^2 + (mx+b)^2 + 2(mx+b) - 24 = 0$$

$$\Leftrightarrow (m^2+1)x^2 + (2bm+2m)x + b^2+2b-24 = 0 \quad \dots(3)$$

Substitusikan (2) ke (3) diperoleh

$$\Leftrightarrow (m^2+1)x^2 + (-2m^2+14m)x + m^2-14m+24 = 0 \quad \dots(4)$$

Syarat menyinggung: $D = 0$

$$D = (-2m^2+14m)^2 - 4(m^2+1)(m^2-14m+24) = 0$$

$$\Leftrightarrow 96m^2 + 56m - 96 = 0 \quad (\text{kedua ruas dibagi 8})$$

$$\Leftrightarrow 12m^2 + 7m - 12 = 0 \Leftrightarrow (4m-3)(3m+4) = 0$$

$$\Leftrightarrow m = \frac{3}{4} \text{ atau } m = -\frac{4}{3}$$

$$\text{Untuk } m = \frac{3}{4} \Rightarrow b = 6 - \frac{3}{4} = \frac{21}{4}$$

$$\text{Jadi, PGS-nya } y = \frac{3}{4}x + \frac{21}{4} \quad \dots(5)$$

Nilai $m = \frac{3}{4}$ substitusikan ke (4) diperoleh

$$\frac{25x^2}{16} + \frac{75x}{8} + \frac{225}{16} = 0 \quad (\text{Kedua ruas dikali } \frac{16}{25})$$

$$\Leftrightarrow x^2 + 6x + 9 = 0 \Leftrightarrow (x+3)^2 = 0 \Leftrightarrow x = -3$$

Substitusikan $x = -3$ ke (5) diperoleh

$$y = \frac{3}{4}(-3) + \frac{21}{4} = 3$$

Titik singgungnya $(-3, 3)$.

Jarak titik $(1, 6)$ ke $(-3, 3)$ adalah

$$\sqrt{(1-(-3))^2 + (6-3)^2} = 5$$

Jawaban: A

35. Pencermian terhadap sumbu $X = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Transformasi oleh $\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ dilanjut pencerminan terhadap sumbu- X ditulis:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ -1 & -2 \end{pmatrix}$$

Sehingga

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-1} \begin{pmatrix} -2 & -5 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2x' + 5y' \\ -x' - 3y' \end{pmatrix}$$

diperoleh: $x = 2x' + 5y'$ dan $y = -x' - 3y'$

Bayangan $x - 2y = 5$ adalah

$$(2x' + 5y') - 2(-x' - 3y') = 5$$

$$2x' + 5y' + 2x' + 6y' = 5$$

$$4x' + 11y' = 5$$

$$\text{atau } 4x + 11y = 5$$

36.

Kelas	Frekuensi
20-29	3
30-39	7
40-49	8
50-59	12
60-69	9
70-79	6
80-89	5

Kelas modus adalah interval 50-59, karena frekuensinya terbesar, yaitu 12.

Dengan demikian,

$$T_b = 49,5, S_1 = 12 - 8 = 4, S_2 = 12 - 9 = 3, \text{ dan } p = 10$$

$$M_o = T_b + \frac{S_1}{S_1 + S_2} \cdot p$$

$$= 49,5 + \frac{4}{4+3} \cdot 10 = 49,5 + \frac{40}{7}$$

Jawaban: D

37.

Berat Badan (kg)	f
36-45	5
46-55	10
56-65	12
66-75	7
76-85	6
$\Sigma = 40$	

Kelas $Q_1 = \frac{1}{4}n = \frac{1}{4}(40) = 10$ terletak pada interval ke-2.

$$T_b = 45,5; F_s = 5; f_{(Q_1)} = 10; l = 10$$

$$Q_1 = T_b + \left(\frac{\frac{1}{4}n - F_s}{f_{(Q_1)}} \right) \cdot l = 45,5 + \frac{10-5}{10} \cdot 10$$

$$= 45,5 + 5 = 50,5$$

Jawaban: A

Jawaban: C

38. "WIYATA" terdiri atas 6 huruf dengan 2 huruf yang sama, yaitu A.

$$\text{Banyak susunan kata} = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 360 \text{ kata}$$

Jawaban: A

$$39. \text{ Banyaknya cara} = {}^8C_3 {}^5C_2 = \frac{8!}{(8-3)!3!} \times \frac{5!}{(5-2)!2!}$$

$$= \frac{8 \times 7 \times 6 \times 5!}{5!3!} \times \frac{5 \times 4 \times 3!}{3!2!}$$

$$= \frac{8 \times 7 \times \cancel{6} \times 5!}{5! \cancel{3} \times 2 \times 1} \times \frac{5 \times \cancel{4} \times 3!}{3! \cancel{2} \times 1}$$

$$= 56 \times 10 = 560$$

Jawaban: D

40. Dalam kantong terdapat 8 kelereng merah dan 10 kelereng putih, akan diambil 2 kelereng sekaligus.

$$n(S) = {}^{18}C_2 = \frac{18!}{2!(18-2)!} = \frac{18 \cdot 17 \cdot 16!}{2 \cdot 1 \cdot 16!} = 153$$

A: terambil 2 kelereng putih dari 10 kelereng

$$n(A) = {}^{10}C_2 = \frac{10!}{2!(10-2)!} = \frac{10 \cdot 9 \cdot 8!}{2 \cdot 1 \cdot 8!} = 45$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{45}{153}$$

Jawaban: C

PEMBAHASAN DAN KUNCI JAWABAN PEMAHAMAN PAKET 4

MATEMATIKA IPA

$$1. \frac{a^{-2}bc^3}{ab^2c^{-1}} = \frac{c^3 \cdot c^1}{a^2 \cdot a \cdot b^2 \cdot b^{-1}} = \frac{c^4}{a^3 \cdot b} = \frac{1^4}{\left(\frac{1}{2}\right)^3 \cdot 2} = \frac{1}{\frac{1}{4}} = 4$$

Jawaban: B

$$2. \frac{8}{3+\sqrt{5}} = \frac{8}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} \\ = \frac{8(3-\sqrt{5})}{3^2-(\sqrt{5})^2} \\ = \frac{8(3-\sqrt{5})}{9-5} \\ = \frac{8(3-\sqrt{5})}{4} \\ = 2(3-\sqrt{5}) \\ = 6-2\sqrt{5}$$

Jawaban: B

$$3. {}^{24}\log 288 = {}^{24}\log (24 \times 12) = {}^{24}\log 24 + {}^{24}\log 12 \\ = 1 + \frac{{}^3\log 12}{{}^3\log 24} = 1 + \frac{{}^3\log (2 \times 6)}{{}^3\log (4 \times 6)} \\ = 1 + \frac{{}^3\log 2 + {}^3\log 6}{{}^3\log 2^2 + {}^3\log 6} \\ = 1 + \frac{q+p}{2q+p} \\ = \frac{(2q+p)+(q+p)}{2q+p} = \frac{3q+2p}{2q+p}$$

Jawaban: A

$$4. \frac{1}{2}\log(x^2-3) - \frac{1}{2}\log x = -1 \\ \frac{1}{2}\log(x^2-3) - \frac{1}{2}\log x = \frac{1}{2}\log\left(\frac{1}{2}\right)^{-1} \\ \frac{1}{2}\log\left(\frac{x^2-3}{x}\right) = \frac{1}{2}\log 2 \\ \frac{x^2-3}{x} = 2 \\ x^2-3 = 2x \\ x^2-2x-3 = 0 \\ (x+1)(x-3) = 0$$

$$x = -1 \text{ atau } x = 3 \quad \dots(1)$$

Syarat numerus:

$$x^2-3 > 0 \text{ dan } x > 0 \quad \dots(2)$$

Jadi, nilai x yang memenuhi x = 3 saja.

Jawaban: E

$$5. (f \circ g)(x) = \frac{1}{x-2} \sqrt{x^2-4x+5} \\ \Leftrightarrow f(g(x)) = \frac{1}{x-2} \sqrt{x^2-4x+5} \\ \Leftrightarrow \sqrt{\{g(x)\}^2+1} = \frac{1}{x-2} \sqrt{x^2-4x+5} \text{ (Kuadratkan kedua ruas)}$$

$$\Leftrightarrow \{g(x)\}^2+1 = \frac{x^2-4x+5}{(x-2)^2}$$

$$\Leftrightarrow \{g(x)\}^2 = \frac{x^2-4x+5}{(x-2)^2} - 1$$

$$\Leftrightarrow \{g(x)\}^2 = \frac{(x^2-4x+5)-(x-2)^2}{(x-2)^2}$$

$$\Leftrightarrow \{g(x)\}^2 = \frac{(x^2-4x+5)-(x^2-4x+4)}{(x-2)^2}$$

$$\Leftrightarrow \{g(x)\}^2 = \frac{x^2-4x+5-x^2+4x-4}{(x-2)^2}$$

$$\Leftrightarrow \{g(x)\}^2 = \frac{1}{(x-2)^2} \Leftrightarrow g(x) = \frac{1}{x-2}$$

$$\Leftrightarrow g(x-3) = \frac{1}{(x-3)-2} = \frac{1}{x-5}, x \neq 5$$

Jawaban: A

$$6. (f \circ g)(x) = f(g(x)) = f(2x-1) = \frac{(2x-1)+4}{(2x-1)-6} = \frac{2x+3}{2x-7}, x \neq \frac{7}{2}$$

Ingat Jika $f(x) = \frac{ax+b}{cx+d}$, maka $f^{-1}(x) = \frac{-dx+b}{cx-a}$

$$(f \circ g)(x) = \frac{2x+3}{2x-7} \Rightarrow (f \circ g)^{-1}(x) = \frac{7x+3}{2x-2}, x \neq 1$$

Jawaban: C

7. Persamaan $x^2 + (m-1)x - 5 = 0$ memiliki akar-akar x_1 dan x_2 , $x_1^2 + x_2^2 - 2x_1x_2 = 8m$.

Kita tentukan dahulu jumlah dan hasil kali akar-akarnya.

$$x_1 + x_2 = -\frac{b}{a} = \frac{-(m-1)}{1} = -m+1$$

$$x_1 \cdot x_2 = \frac{c}{a} = \frac{-5}{1} = -5$$

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 \\ = (-m+1)^2 - 2(-5) \\ = m^2 - 2m + 1 + 10 \\ = m^2 - 2m + 11$$

$$\begin{aligned}
 x_1^2 + x_2^2 - 2x_1x_2 &= 8m \\
 (m^2 - 2m + 11) - 2(-5) &= 8m \\
 m^2 - 2m + 11 + 10 - 8m &= 0 \\
 m^2 - 10m + 21 &= 0 \\
 (m - 3)(m - 7) &= 0 \\
 m &= 3 \text{ atau } m = 7
 \end{aligned}$$

Jawaban: B

8. $3x^2 + 5x - 2 = 0$, akar-akarnya x_1 dan x_2

$$x_1 + x_2 = \frac{-b}{a} = -\frac{5}{3}$$

$$x_1 \cdot x_2 = \frac{c}{a} = -\frac{2}{3}$$

Persamaan kuadrat yang akar-akarnya $2x_1 - 3$ dan $2x_2 - 3$.

Misal:

$$b = 2x_1 - 3$$

$$c = 2x_2 - 3$$

$$\begin{aligned}
 b + c &= 2x_1 - 3 + 2x_2 - 3 = 2(x_1 + x_2) - 6 \\
 &= 2\left(-\frac{5}{3}\right) - 6 = -\frac{10}{3} - 6 = -\frac{28}{3}
 \end{aligned}$$

$$\begin{aligned}
 b \cdot c &= (2x_1 - 3)(2x_2 - 3) \\
 &= 4x_1x_2 - 6x_1 - 6x_2 + 9 \\
 &= 4(x_1x_2) - 6(x_1 + x_2) + 9 \\
 &= 4\left(-\frac{2}{3}\right) - 6\left(-\frac{5}{3}\right) + 9 \\
 &= -\frac{8}{3} + \frac{30}{3} + 9 = \frac{22}{3} + 9 = \frac{49}{3}
 \end{aligned}$$

Persamaan kuadrat baru:

$$\begin{aligned}
 x^2 - (b + c)x + b \cdot c &= 0 \\
 x^2 - \left(-\frac{28}{3}\right)x + \left(\frac{49}{3}\right) &= 0 \\
 3x^2 + 28x + 49 &= 0
 \end{aligned}$$

Jawaban: D

9. Premis (1): Jika hari hujan, maka ibu memakai payung.

Premis (2): Ibu tidak memakai payung.

Kedua premis di atas dapat disimpulkan sebagai berikut.

$$\begin{array}{l}
 P_1 : p \Rightarrow q \\
 P_2 : \sim q \\
 \hline
 \therefore \sim p \quad (\text{Modus Tollens})
 \end{array}$$

Jadi, kesimpulan yang sah adalah hari tidak hujan.

Jawaban: A

10. Grafik $y = px^2 + (p + 2)x - p + 4$ memotong sumbu-X di dua titik.

Grafik memotong sumbu-X di dua titik, maka $D > 0$

$$D > 0 \Leftrightarrow b^2 - 4ac > 0$$

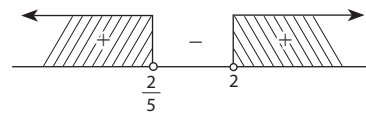
$$(p + 2)^2 - 4p(-p + 4) > 0$$

$$p^2 + 4p + 4 + 4p^2 - 16p > 0$$

$$5p^2 - 12p + 4 > 0$$

$$(5p - 2)(p - 2) > 0$$

Pembuat nol fungsi, $p = \frac{2}{5}$ dan $p = 2$



$$p < \frac{2}{5} \text{ atau } p > 2$$

Jadi, batas-batas nilai p yang memenuhi adalah

$$p < \frac{2}{5} \text{ atau } p > 2.$$

Jawaban: B

$$\begin{array}{rcl}
 11. & 2A + 3B = 14.000 & \times 3 \quad 6A + 9B = 42.000 \\
 & 3A + 4B = 19.000 & \times 2 \quad 6A + 8B = 38.000 \\
 & & \hline
 & & B = 4.000
 \end{array}$$

$$B = 4.000 \Rightarrow 2A + 3(4.000) = 14.000$$

$$2A + 12.000 = 14.000$$

$$2A = 14.000 - 12.000$$

$$2A = 2.000$$

$$A = 1.000$$

$$A + B = 1.000 + 4.000 = 5.000$$

$$\text{Uang kembalian} = 10.000 - 5.000 = 5.000$$

Jadi, Nova akan menerima uang kembalian sebesar Rp5.000,00.

Jawaban: E

12. Misalkan banyak mobil = x dan banyak bus = y

Jenis	Luas (m ²)	Daya Muat	Biaya Parkir
Mobil	4	x	100
Bus	20	y	200
	176	20	

Model matematika:

$$x + y \leq 20,$$

$$4x + 20y \leq 176 \Leftrightarrow x + 5y \leq 44,$$

$$x \geq 0, y \geq 0$$

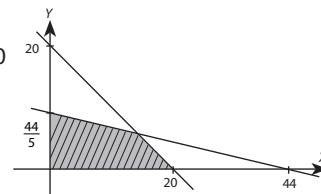
Fungsi tujuan: maksimalkan $f(x, y) = 100x + 200y$

Sketsa gambar:

Titik potong $x + y = 20$

dan $x + 5y = 44$

$$\begin{array}{rcl}
 x + y & = & 20 \\
 x + 5y & = & 44 \\
 \hline
 -4y & = & -24 \\
 y & = & 6 \Rightarrow x = 14
 \end{array}$$



Titik potongnya (14, 6)

Titik Pojok	$f(x, y) = 100x + 200y$
(20, 0)	2.000
(14, 6)	$1.400 + 1.200 = 2.600$
$(0, \frac{44}{5})$	1.760

→ maksimum

Jadi, penghasilan maksimumnya Rp2.600,00.

Jawaban: D

$$13. \begin{pmatrix} 5 & -2 \\ 9 & -4 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ x & x+y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 10-2x & -2x-2y-5 \\ 18-4x & -4x-4y-9 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Diperoleh

$$\begin{aligned} \bullet \quad 10-2x &= 1 & \bullet \quad -2x-2y-5 &= 0 \\ -2x &= -9 & -2\left(\frac{9}{2}\right)-2y-5 &= 0 \\ x &= \frac{9}{2} & -9-2y-5 &= 0 \\ & & -2y &= 14 \\ & & y &= -7 \end{aligned}$$

$$\text{Sehingga, } x-y = \frac{9}{2} - (-7) = \frac{9}{2} + \frac{14}{2} = \frac{23}{2}$$

Jawaban: E

$$14. A = \begin{pmatrix} \frac{4}{7} & -\frac{1}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{pmatrix}^{-1} = \frac{1}{\frac{8}{49} - \frac{1}{49}} \begin{pmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{1}{7} & \frac{4}{7} \end{pmatrix}$$

$$= 7 \begin{pmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{1}{7} & \frac{4}{7} \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$$

$$A^T B = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ 2 & 8 \end{pmatrix} = \begin{pmatrix} 10 & 12 \\ 12 & 34 \end{pmatrix}$$

$$\det A^T B = (10)(34) - (12)(12) = 196$$

Jawaban: D

15. Deret aritmetika dengan $S_n = n^2 + 3n$.

Dalam deret aritmetika berlaku: $U_n = S_n - S_{n-1}$

$$\begin{aligned} \text{Sehingga, } U_{20} &= S_{20} - S_{19} \\ &= (20^2 + 3(20)) - (19^2 + 3(19)) \\ &= (400 + 60) - (361 + 57) \\ &= 460 - 418 = 42 \end{aligned}$$

Jawaban: B

16. Misalkan uang yang diterima si bungsu = x, maka:

anak ke-3 memperoleh $x + 5.000$

anak ke-2 memperoleh $x + 10.000$

anak ke-1/si sulung memperoleh $x + 15.000$

$$(x + 15.000) + (x + 10.000) + (x + 5.000) + x = 100.000$$

$$4x + 30.000 = 100.000$$

$$4x = 100.000 - 30.000$$

$$4x = 70.000$$

$$x = 17.500$$

Jadi, uang yang diterima si bungsu adalah Rp17.500,00.

Jawaban: B

$$17. U_5 = ar^4 = 16r^4 = 81$$

$$r^4 = \frac{81}{16} \Rightarrow r = \frac{3}{2}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_5 = \frac{16\left(\left(\frac{3}{2}\right)^5 - 1\right)}{\frac{3}{2} - 1} = \frac{16\left(\frac{243}{32} - 1\right)}{\frac{1}{2}} = \frac{16\left(\frac{211}{32}\right)}{\frac{1}{2}}$$

$$= \frac{211}{2} \times \frac{2}{1} = 211$$

Jadi, panjang tali semula adalah 211 cm.

Jawaban: B

$$18. P(x) = 2x^4 + ax^3 - 3x^2 + 5x + b$$

$P(x)$ dibagi $(x - 1)$ sisa 11, artinya $P(1) = 11$

$$P(1) = 2(1)^4 + a(1)^3 - 3(1)^2 + 5(1) + b = 11$$

$$2 + a - 3 + 5 + b = 11$$

$$a + b = 7 \quad \dots (1)$$

$P(x)$ dibagi $(x + 1)$ sisa -1, artinya $P(-1) = -1$

$$P(-1) = 2(-1)^4 + a(-1)^3 - 3(-1)^2 + 5(-1) + b = -1$$

$$2 - a - 3 - 5 + b = -1$$

$$-a + b = 5 \quad \dots (2)$$

$$a + b = 7$$

$$\frac{-a + b = 5}{2b = 12} \Rightarrow b = 6$$

Untuk $b = 6$, maka $a + b = 7 \Leftrightarrow a + 6 = 7 \Leftrightarrow a = 1$

Jadi, nilai $2a + b = 2(1) + 6 = 8$.

Jawaban: C

$$19. \lim_{x \rightarrow 0} \frac{5x}{3 - \sqrt{9+x}} = \lim_{x \rightarrow 0} \frac{5x}{3 - \sqrt{9+x}} \times \frac{3 + \sqrt{9+x}}{3 + \sqrt{9+x}}$$

$$= \lim_{x \rightarrow 0} \frac{5x(3 + \sqrt{9+x})}{3^2 - (\sqrt{9+x})^2} = \lim_{x \rightarrow 0} \frac{5x(3 + \sqrt{9+x})}{9 - (9+x)}$$

$$= \lim_{x \rightarrow 0} \frac{5x(3 + \sqrt{9+x})}{-x}$$

$$= \lim_{x \rightarrow 0} -5(3 + \sqrt{9+x})$$

$$= -5(3 + \sqrt{9+0}) = -5(3+3) = -30$$

Jawaban: A

20. Jika $\lim_{x \rightarrow \infty} \{(bx+c) - \sqrt{ax^2+px+q}\} = L$,

maka berlaku $L = \frac{2bc-p}{2\sqrt{a}}$.

Sehingga,

$$\begin{aligned}\lim_{x \rightarrow \infty} \{(3x-2) - \sqrt{9x^2-2x+5}\} \\&= \frac{2(3)(-2) - (-2)}{2\sqrt{9}} \\&= \frac{-12+2}{2(3)} = \frac{-10}{6} = -\frac{5}{3}\end{aligned}$$

Jawaban: E

21. $f(x) = \frac{1}{3}x^3 + 3x^2 - 16x + 1$

$f'(x) = x^2 + 6x - 16$

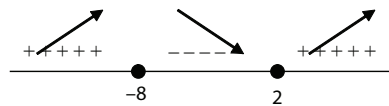
$f''(x) = 2x + 6$

Nilai stasioner jika $f'(x) = 0$

$x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

$x = -8$ atau $x = 2$



Berdasarkan garis bilangan di atas dapat diperoleh fungsi f mencapai nilai maksimum relatif untuk $x = -8$.

Sehingga,

$$\begin{aligned}f(-8) &= \frac{1}{3}(-8)^3 + 3(-8)^2 - 14(-8) + 1 \\&= \frac{1}{3}(-512) + 3(64) - 14(-8) + 1 \\&= -\frac{512}{3} + 192 + 112 + 1 \\&= -170\frac{2}{3} + 305 = 150\frac{1}{3}\end{aligned}$$

Jawaban: B

22. $\int \frac{2x+3}{\sqrt{3x^2+9x-1}} dx$

Misalkan $u = 3x^2 + 9x - 1 \Rightarrow \frac{du}{dx} = 6x + 9 = 3(2x+3)$
 $\frac{du}{3} = (2x+3)dx$

$$\begin{aligned}\int \frac{2x+3}{\sqrt{3x^2+9x-1}} dx &= \int \frac{1}{\sqrt{u}} \cdot \frac{du}{3} = \frac{1}{3} \int \frac{1}{\sqrt{u}} du \\&= \frac{1}{3} \int u^{-\frac{1}{2}} du \\&= \frac{2}{3} u^{\frac{1}{2}} + C = \frac{2}{3} \sqrt{3x^2+9x-1} + C\end{aligned}$$

Jawaban: C

23. $\int_0^{\frac{1}{2}\pi} (2\sin 2x - 3\cos x) dx = 2 \cdot \frac{1}{2} (-\cos 2x) - 3 \sin x \Big|_0^{\frac{1}{2}\pi}$
 $= \left[-\cos(\pi) - 3\sin\left(\frac{1}{2}\pi\right) \right] - \left[-\cos 0 - 3\sin 0 \right]$
 $= 1 - 3(1) + 1 + 0 = -1$

Jawaban: B

24. Luas kotak tanpa tutup yang alasnya persegi = 432 cm^2

Misal rusuk persegi = $x \text{ cm}$ dan rusuk yang lain = tinggi kotak = t .

$L = x^2 + 4xt = 432$

$t = \frac{432 - x^2}{4x}$

$r = x^2 t = x^2 \left(\frac{432 - x^2}{4x} \right) = \frac{432x - x^3}{4}$

r maksimum apabila $r' = 0$

$\frac{432}{4} - \frac{3}{4}x^2 = 0$

$\frac{3}{4}x^2 = \frac{432}{4}$

$x^2 = 144$

$x = 12$

Jawaban: D

25. $A + B = \frac{\pi}{2}; \sin A \sin B = \frac{1}{4}$

$\cos(A+B) = \cos \frac{\pi}{2} = 0$

$\cos A \cos B - \sin A \sin B = 0$

$\cos A \cos B - \frac{1}{4} = 0$

$\cos A \cos B = \frac{1}{4}$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Jawaban: C

26. $\frac{\cos 235^\circ - \cos 125^\circ}{\sin 235^\circ - \sin 125^\circ}$

$= \frac{-2\sin\left(\frac{235^\circ+125^\circ}{2}\right)\sin\left(\frac{235^\circ-125^\circ}{2}\right)}{\sin 235^\circ - \sin 125^\circ}$

$= \frac{-2\sin 180^\circ \sin 55^\circ}{\sin 235^\circ - \sin 125^\circ}$

$= \frac{-2 \cdot 0 \cdot \sin 55^\circ}{\sin 235^\circ - \sin 125^\circ} = 0$

Jawaban: A

27. $\cos 4x + 3\sin 2x = -1, 0^\circ \leq x \leq 180^\circ$

$(1 - 2\sin^2 2x) + 3\sin 2x = -1$

$-2(\sin 2x)^2 + 3(\sin 2x) + 1 + 1 = 0$

$-2(\sin 2x)^2 + 3\sin 2x + 2 = 0$

$(-2\sin 2x - 1)(\sin 2x - 2) = 0$

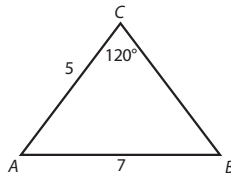
$\sin 2x = -\frac{1}{2}$ atau $\sin 2x = 2$ (tidak memenuhi)

- $\sin 2x = \sin 210^\circ$ $\sin 2x = \sin 330^\circ$
- $x = 105^\circ$ $x = 165^\circ$

Jadi, himpunan penyelesaian $\{105^\circ, 165^\circ\}$

Jawaban: E

28.



Aturan kosinus

$$AB^2 = AC^2 + BC^2 - 2AC \cdot BC \cos C$$

$$7^2 = 5^2 + BC^2 - 2 \cdot 5 \cdot BC \cos 120^\circ$$

$$49 = 25 + BC^2 - 10BC \left(-\frac{1}{2}\right)$$

$$24 = BC^2 + 5BC$$

$$BC^2 + 5BC - 24 = 0$$

$$(BC + 8)(BC - 3) = 0$$

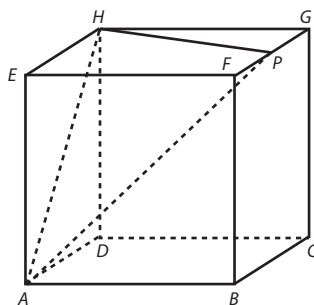
$$BC = -8 \text{ (tidak memenuhi) atau } BC = 3$$

Jadi, keliling $\triangle ABC = AB + BC + AC$

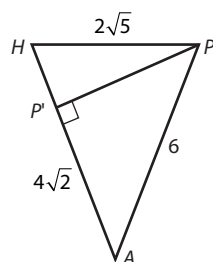
$$= 7 \text{ cm} + 3 \text{ cm} + 5 \text{ cm} = 15 \text{ cm}$$

Jawaban: B

29. Perhatikan gambar berikut.



Perhatikan $\triangle APH$.



$$PH = \sqrt{GH^2 + PG^2}$$

$$= \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$$

$$BP = PH = 2\sqrt{5}$$

$$AP = \sqrt{AB^2 + BP^2}$$

$$= \sqrt{4^2 + (2\sqrt{5})^2} = 6$$

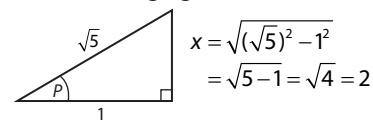
$$AH^2 = AP^2 + PH^2 - 2 \cdot AP \cdot PH \cos P$$

$$(4\sqrt{2})^2 = 6^2 + (2\sqrt{5})^2 - 2 \cdot 6 \cdot 2\sqrt{5} \cos P$$

$$32 = 36 + 20 - 24\sqrt{5} \cos P$$

$$\cos P = \frac{24}{24\sqrt{5}} = \frac{1}{\sqrt{5}}$$

Perhatikan segitiga berikut.



$$\text{Sehingga, } \sin P = \frac{2}{\sqrt{5}}$$

$$\text{Luas } \triangle APH = \frac{1}{2} \cdot AP \cdot PH \sin P$$

$$= \frac{1}{2} \cdot 6 \cdot 2\sqrt{5} \cdot \frac{2}{\sqrt{5}} = 12$$

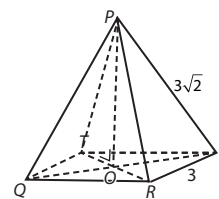
$$\text{Luas } \triangle APH = \frac{1}{2} \cdot AH \cdot PP'$$

$$12 = \frac{1}{2} \cdot 4\sqrt{2} \cdot PP'$$

$$PP' = \frac{12}{2\sqrt{2}} = 3\sqrt{2} \text{ cm}$$

Jawaban: D

30. Limas $P.QRST$ dengan rusuk alas 3 cm dan rusuk tegak $3\sqrt{2}$ cm.



Perhatikan $\triangle PTO$

$$TO = \frac{1}{2} TR = \frac{1}{2} \text{ diagonal} = \frac{1}{2} (3\sqrt{2}) = \frac{3}{2} \sqrt{2} \text{ cm}$$

Perhatikan $\triangle POS$

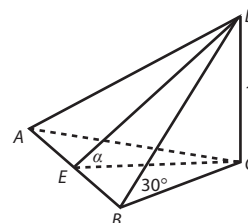
$$PO = \sqrt{PS^2 - OS^2} = \sqrt{(3\sqrt{2})^2 - \left(\frac{3}{2}\sqrt{2}\right)^2}$$

$$= \sqrt{18 - \frac{9}{2}} = \sqrt{\frac{27}{2}} = \sqrt{\frac{54}{4}} = \frac{3}{2} \sqrt{6} \text{ cm}$$

$$\tan \angle (PT, QRST) = \frac{PO}{TO} = \frac{\frac{3}{2} \sqrt{6}}{\frac{3}{2} \sqrt{2}} = \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$$

Jawaban: C

31.



Perhatikan $\triangle BCD$.

$$\sin 30^\circ = \frac{CD}{BD} \Leftrightarrow BD = \frac{CD}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2$$

$$\tan 30^\circ = \frac{CD}{BC} \Leftrightarrow BC = \frac{CD}{\tan 30^\circ} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

Karena ABC segitiga samasisi, maka $AB = BC = \sqrt{3}$.

$$BE = \frac{1}{2} AB = \frac{1}{2} \sqrt{3}$$

Perhatikan $\triangle BED$ siku-siku di E .

$$DE = \sqrt{BD^2 - BE^2} = \sqrt{2^2 - \left(\frac{1}{2}\sqrt{3}\right)^2} = \frac{1}{2}\sqrt{13}$$

Perhatikan $\triangle CED$ siku-siku di C .

$$EC = \sqrt{DE^2 - CD^2} = \sqrt{\left(\frac{1}{2}\sqrt{13}\right)^2 - 1^2} = \frac{3}{2}$$

$$\tan \alpha = \frac{CD}{EC} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

Jawaban: E

32. $\vec{a} \perp \vec{b}$, maka $\vec{a} \cdot \vec{b} = 0$

$$\begin{pmatrix} p \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix} = 0$$

$$4p - 6 - 6 = 0$$

$$4p = 12$$

$$p = 3$$

$$\begin{aligned} (\vec{a} - 2\vec{b}) \cdot (3\vec{c}) &= \left(\begin{pmatrix} p \\ 2 \\ -1 \end{pmatrix} - 2 \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix} \right) \cdot 3 \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} p-8 \\ 8 \\ -13 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -3 \\ 9 \end{pmatrix} = 6(p-8) - 24 - 13(9) \\ &= 6(3-8) - 141 = -30 - 141 = -171 \end{aligned}$$

Jawaban: E

33. Pusat $(-1, -3)$ dan menyinggung garis $y = 2$, maka $r = 5$

Persamaan lingkaran:

$$(x+1)^2 + (y+3)^2 = 5^2$$

$$x^2 + 2x + 1 + y^2 + 6y + 9 = 25$$

$$x^2 + y^2 + 2x + 6y - 15 = 0$$

Jawaban: B

34. Lingkaran $L \equiv (x+1)^2 + (y-3)^2 = 9$ memotong garis $y = 3$.

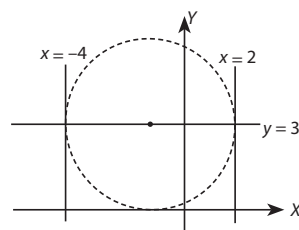
Garis singgung yang melalui titik potong L dan garis.

Titik potong lingkaran dengan garis

$$y = 3 \Rightarrow (x+1)^2 + (3-3)^2 = 9$$

$$(x+1)^2 = 9$$

$$x = 2 \text{ atau } x = -4$$



Garis singgungnya adalah $x = 2$ dan $x = -4$

Jawaban: A

35. Rotasi dengan pusat $O(0, 0)$ sejauh 90° .

$$R(0, 90^\circ) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Dilatasi dengan pusat $O(0, 0)$ dan $k = 3 = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

$R(0, 90^\circ)$ dilanjutkan dilatasi $[O, 3]$ adalah

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix}$$

$$\text{Sehingga } \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3y \\ 3x \end{pmatrix}$$

$$\text{Diperoleh: } x' = -3y \Rightarrow y = -\frac{1}{3}x'$$

$$y' = 3x \Rightarrow x = \frac{1}{3}y'$$

$$\begin{aligned} y = 3x - 9x^2 &\xrightarrow{T_2 \circ T_1} -\frac{1}{3}x = 3\left(\frac{1}{3}y'\right) - 9\left(\frac{1}{3}y'\right)^2 \\ -\frac{1}{3}x &= y - y^2 \\ x &= 3y^2 - 3y \end{aligned}$$

Jawaban: A

36. Data dengan frekuensi terbanyak terdapat di kelas ke-3 (60-64)

$$T_b = 60 - 0,5 = 59,5$$

$$l = 5$$

$$s_1 = 14 - 12 = 2$$

$$s_2 = 14 - 7 = 7$$

$$Mo = T_b + \frac{s_1}{s_1 + s_2} \cdot l$$

$$= 59,5 + \frac{2}{2+7} \cdot 5 = 59,5 + \frac{10}{9}$$

$$= 59,5 + 1,1 = 60,6$$

Jawaban: A

37.

Berat badan (kg)	f
36-45	5
46-55	10
56-65	12
66-75	7
76-85	6
$\Sigma = 40$	

Kelas $Q_1 = \frac{1}{4}n = \frac{1}{4}(40) = 10$ terletak pada interval ke-2.

$$T_b = 45,5; F_s = 5; f_{(Q_1)} = 10; l = 10$$

$$Q_1 = T_b + \left(\frac{\frac{1}{4}n - F_s}{f_{(Q_1)}} \right) \cdot l = 45,5 + \frac{10 - 5}{10} \cdot 10 = 45,5 + 5 = 50,5$$

Jawaban: A

38. Siswa wajib mengerjakan 8 dari 10 soal, tetapi 1 sampai 4 wajib dikerjakan.

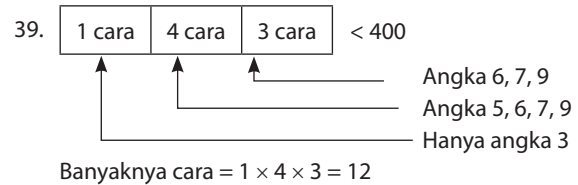
Karena 4 soal wajib dikerjakan, maka ada 4 soal lagi yang harus dipilih dari 6 soal sisa, yang berarti kombinasi 4 dari 6.

$$C_{(n,r)} = \frac{n!}{r!(n-r)!}$$

$$C_{(6,4)} = \frac{6!}{4!(6-4)!} = \frac{6 \cdot 5 \cdot 4!}{4! \cdot 2 \cdot 1} = \frac{30}{2} = 15$$

Jadi, ada 15 cara untuk memilih soal.

Jawaban: B



Jawaban: B

40. Pelemparan 2 buah dadu bersama-sama satu kali.

$$n(S) = 36$$

$A =$ muncul mata dadu berjumlah 5 = $\{(1, 4), (2, 3), (3, 2), (4, 1)\}$, $n(A) = 4$

$B =$ muncul mata dadu berjumlah 7 = $\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$, $n(B) = 6$

$$P(A \cup B) = P(A) + P(B) = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} = \frac{4}{36} + \frac{6}{36} = \frac{10}{36} = \frac{5}{18}$$

Jawaban: C

PEMBAHASAN DAN KUNCI JAWABAN PEMAHAMAN PAKET 5

MATEMATIKA IPA

1. $(a^{-1})^2 \times \frac{b^4}{c^{-3}} = \frac{b^4 \cdot c^3}{a^2} = \frac{2^4 \cdot \left(\frac{1}{2}\right)^3}{4^2} = \frac{2}{16} = \frac{1}{8}$

Jawaban: C

2.
$$\frac{\sqrt{5} + 2\sqrt{3}}{\sqrt{5} - 3\sqrt{3}} = \frac{\sqrt{5} + 2\sqrt{3}}{\sqrt{5} - 3\sqrt{3}} \times \frac{\sqrt{5} + 3\sqrt{3}}{\sqrt{5} + 3\sqrt{3}} = \frac{(\sqrt{5} + 2\sqrt{3})(\sqrt{5} + 3\sqrt{3})}{(\sqrt{5})^2 - (3\sqrt{3})^2} = \frac{5 + 5\sqrt{15} + 18}{5 - 27} = \frac{23 + 5\sqrt{15}}{-22}$$

Jawaban: E

3.
$$\begin{aligned} & {}^{3 \times 3} \log(8 \times 4) \cdot {}^2 \log \sqrt{81} - \sqrt{4} \log(5 \times 5) \cdot {}^5 \log 4 \\ &= {}^9 \log 3^2 \cdot {}^2 \log 9 - {}^2 \log 25 \cdot {}^5 \log 2^2 \\ &= {}^{3^2} \log 2^5 \cdot {}^2 \log 3^2 - {}^2 \log 5^2 \cdot {}^5 \log 2^2 \\ &= \frac{1}{2} \cdot 5 \cdot 2 - 2 \cdot 2 = 5 - 4 = 1 \end{aligned}$$

Jawaban: E

4. $9^{2x} - 10 \cdot 9^x + 9 > 0, x \in R$
 $(9^x)^2 - 10 \cdot 9^x + 9 > 0$

Misal: $y = 9^x$, maka persamaannya menjadi:

$$y^2 - 10y + 9 > 0$$

$$(y - 1)(y - 9) > 0$$

Pembuat nol fungsi $y = 1$ dan $y = 9$



$$y < 1 \quad \text{atau} \quad y > 9$$

$$9^x < 1 \quad \text{atau} \quad 9^x > 9$$

$$9^x < 9^0 \quad \text{atau} \quad 9^x > 9^1$$

$$x < 0 \quad \text{atau} \quad x > 1$$

Jadi, nilai x yang memenuhi adalah $x < 0$ atau $x > 1$

Jawaban: B

5. $f(x) = 2x - 3$ dan $g(x) = x^2 + 2x - 3$

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) = g(2x - 3) \\ &= (2x - 3)^2 + 2(2x - 3) - 3 \\ &= 4x^2 - 12x + 9 + 4x - 6 - 3 \\ &= 4x^2 - 8x \end{aligned}$$

Jawaban: E

6. $f(x) = \frac{3x+2}{2x-3}, x \neq \frac{3}{2}$ dan $h(x) = 5x+1$

$$\begin{aligned}(f \circ h)(x) &= f(h(x)) = f(5x+1) \\ &= \frac{3(5x+1)+2}{2(5x+1)-3} = \frac{15x+3+2}{10x+2-3} \\ &= \frac{15x+5}{10x-1}, x \neq \frac{1}{10}\end{aligned}$$

Jika $f(x) = \frac{ax+b}{cx+d}$, maka $f^{-1}(x) = \frac{-dx+b}{cx-a}$

$$\begin{aligned}(f \circ h)(x) &= \frac{15x+5}{10x-1}, x \neq \frac{1}{10} \\ \Rightarrow (f \circ h)^{-1}(x) &= \frac{x+5}{10x-15}, x \neq \frac{3}{2}\end{aligned}$$

Jawaban: B

7. $x_1 + x_2 = -\frac{(-8)}{1} = 8$ } ingat rumus jumlah dan
hasil kali akar-akar

$$x_1 \cdot x_2 = \frac{c}{1} = c$$

$$x_1 + x_2 = 8 \Leftrightarrow x_1 + 3x_1 = 8$$

$$4x_1 = 8$$

$$x_1 = 2$$

$$x_2 = 8 - x_1 = 8 - 2 = 6$$

$$x_1 \cdot x_2 = c \Leftrightarrow 2(6) = c \Leftrightarrow c = 12$$

Jawaban: D

8. Persamaan $2x^2 - 2(p-4)x + p = 0$ mempunyai dua akar real berbeda, berarti $D > 0$

$$b^2 - 4ac > 0$$

$$(-2p+8)^2 - 4(2)(p) > 0$$

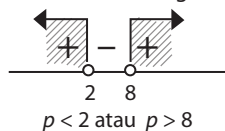
$$4p^2 - 32p + 64 - 8p > 0$$

$$4p^2 - 40p + 64 > 0$$

$$p^2 - 10p + 16 > 0$$

$$(p-2)(p-8) > 0$$

Pembuat nol fungsi adalah $p = 2, p = 8$



Jawaban: B

9. "Jika semua anggota keluarga pergi, maka semua pintu rumah dikunci rapat"

Pernyataan di atas merupakan suatu implikasi yang mengandung pernyataan berkuantor (mengandung kata semua)

Ingat: $\sim(p \Rightarrow q) \equiv p \wedge \sim q$ dan

$\sim(\text{semua } q) \equiv \text{ada } \sim q$

Dengan demikian, negasi pernyataan tersebut adalah:
"Semua anggota keluarga pergi dan ada pintu rumah yang tidak dikunci rapat."

Jawaban: D

10. Grafik fungsi $y = x^2 - ax + 6 + 2a$ menyinggung garis $y = x - 1$.

$$y = y$$

$$x^2 - ax + 6 + 2a = x - 1$$

$$x^2 - ax - x + 6 + 2a + 1 = 0$$

$$x^2 - (a+1)x + 7 + 2a = 0$$

Dengan $a = 1, b = -(a+1)$, dan $c = 7 + 2a$

Karena menyinggung, maka $D = 0$

$$D = 0$$

$$b^2 - 4ac = 0$$

$$(-(a+1))^2 - 4(1)(7+2a) = 0$$

$$a^2 + 2a + 1 - 28 - 8a = 0$$

$$a^2 - 6a - 27 = 0$$

$$(a-9)(a+3) = 0$$

$$a = 9 \text{ atau } a = -3$$

Jawaban: B

11. Misalkan harga 1 meter sutra = x dan harga 1 meter katun = y

maka: $x = 3y$

$$5x + 4y = 228.000$$

$$5(3y) + 4y = 228.000$$

$$15y + 4y = 228.000$$

$$19y = 228.000$$

$$y = 12.000$$

Diperoleh $x = 3(12.000) = 36.000$

Jadi, harga 1 meter sutra adalah Rp36.000,00.

Jawaban: B

12. Misalkan: tablet I = x , tablet II = y

Tablet	Vit. A	Vit. B	Harga
I	5	3	4.000
II	10	1	8.000
	25	5	

Fungsi tujuan:

$$\text{minimumkan } f(x, y) = 4.000x + 8.000y$$

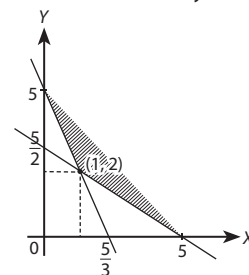
Model matematikanya:

$$x \geq 0; y \geq 0; 5x + 10y \geq 25; 3x + y \geq 5$$

Titik potong pada kedua garis

$$\begin{aligned}5x + 10y &= 25 \Leftrightarrow x + 2y = 5 \quad \times 1 \\ 3x + y &= 5 \quad \times 2 \quad \frac{6x + 2y = 10}{-5x = -5} \\ x &= 1\end{aligned}$$

untuk $x = 1$, maka $y = 2$



Titik potongnya (1, 2)

Titik Pojok	$f(x, y) = (4.000x + 8.000y)$
(5, 0)	$4.000(5) + 0 = 20.000$
(1, 2)	$4.000(1) + 8.000(2) = 20.000$
(0, 5)	$0 + 8.000(5) = 40.000$

Jadi, pengeluaran minimumnya adalah Rp20.000,00.

Jawaban: E

$$13. \quad A+B-C = \begin{pmatrix} 3 & y \\ 5 & -1 \end{pmatrix} + \begin{pmatrix} x & 5 \\ -3 & 6 \end{pmatrix} - \begin{pmatrix} -3 & -1 \\ y & 9 \end{pmatrix}$$

$$\begin{pmatrix} 8 & 5x \\ -x & -4 \end{pmatrix} = \begin{pmatrix} 3+x+3 & y+5+1 \\ 5-3-y & -1+6-9 \end{pmatrix}$$

$$\begin{pmatrix} 8 & 5x \\ -x & -4 \end{pmatrix} = \begin{pmatrix} 6+x & y+6 \\ 2-y & -4 \end{pmatrix}$$

Diperoleh:

$$8 = 6 + x \Rightarrow x = 2$$

$$5x = y + 6 \Rightarrow 10 = y + 6 \Rightarrow y = 4$$

$$x + 2xy + y = 2 + 2(2)(4) + 4 = 2 + 16 + 4 = 22$$

Jawaban: E

$$14. \quad A = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}, B = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}, \text{ dan } C = A - 2B$$

$$C = A - 2B = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} - 2 \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} -2 & 2 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 0 & -5 \end{pmatrix}$$

C^{-1} adalah invers dari matriks C , maka

$$C^{-1} = \frac{1}{(4)(-5) - (0)(-1)} \begin{pmatrix} -5 & 1 \\ 0 & 4 \end{pmatrix} = \frac{1}{-20} \begin{pmatrix} -5 & 1 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-5}{-20} & \frac{1}{-20} \\ \frac{0}{-20} & \frac{4}{-20} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{20} \\ 0 & -\frac{1}{5} \end{pmatrix}$$

Jawaban: D

$$15. \quad \text{Barisan geometri, } U_2 = \frac{4}{3} \text{ dan } U_5 = 36.$$

$$U_2 = ar = \frac{4}{3}$$

$$U_5 = ar^4 = 36$$

$$\frac{U_5}{U_2} = \frac{ar^4}{ar} = \frac{36}{\frac{4}{3}}$$

$$r^3 = 27$$

$$r = 3$$

$$U_6 = ar^5 = U_5 \cdot r = 36 \cdot 3 = 108$$

Jawaban: A

$$16. \quad \text{Dalam deret aritmetika, } S_n = \frac{n}{2} (2a + (n-1)b)$$

Sehingga:

$$S_{10} = \frac{10}{2} (2(1.600.000) + 9(200.000))$$

$$= 5(3.200.000 + 1.800.000)$$

$$= 5(5.000.000) = 25.000.000$$

Jadi, total seluruh gaji yang diterima adalah Rp25.000.000,00.

Jawaban: C

$$17. \quad \text{Barisan geometri dengan } n = 7, U_1 = a = 6 \text{ dan } U_7 = 384.$$

$$U_7 = ar^6 = 6r^6 = 384$$

$$r^6 = 64$$

$$r = 2$$

$$\text{Jumlah deret geometri dirumuskan: } S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_7 = \frac{6(2^7 - 1)}{2 - 1} = \frac{6(128 - 1)}{1} = 762$$

Jadi, panjang keseluruhan tali tersebut adalah 762 cm.

Jawaban: D

$$18. \quad \text{Misalkan: } h(x) \text{ dibagi } (x^2 - 2x - 3) \text{ bersisa } Ax + B$$

$$h(x) = k(x)(x - 3)(x + 1) + Ax + B$$

$$h(3) = 0 + A(3) + B = f(3)g(3)$$

$$= 4 \cdot 15 = 60 \Rightarrow 3A + B = 60$$

$$h(-1) = 0 + A(-1) + B = f(-1)g(-1)$$

$$= 8 \cdot (-9) = -72 \Rightarrow -A + B = -72$$

$$4A = 132$$

$$A = 33$$

$$\text{Sehingga, } B = -72 + 33 = -39$$

Jadi, sisa pembagiannya $33x - 39$.

Jawaban: E

$$19. \quad \lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 2x} = \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{x(x-2)}$$

$$= \lim_{x \rightarrow 2} \frac{x^2 + 2x + 4}{x} = \frac{2^2 + 2 \cdot 2 + 4}{2} = \frac{12}{2} = 6$$

Apabila menggunakan dalil L'Hospital, maka:

$$\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 2x} \stackrel{L}{=} \lim_{x \rightarrow 2} \frac{3x^2}{2x - 2} \left(\begin{array}{l} \text{pembilang dan penyebut,} \\ \text{masing-masing diturunkan} \end{array} \right)$$

$$= \frac{3(2)^2}{2(2) - 2} = \frac{12}{2} = 6$$

Jawaban: D

$$20. \quad \lim_{x \rightarrow \infty} (\sqrt{x-1} - \sqrt{x+2})(\sqrt{x-3})$$

$$= \lim_{x \rightarrow \infty} (\sqrt{(x-1)(x-3)} - \sqrt{(x+2)(x-3)})$$

$$= \lim_{x \rightarrow \infty} (\sqrt{x^2 - 4x + 3} - \sqrt{x^2 - x - 6})$$

$$= \frac{-4 - (-1)}{2\sqrt{1}} = \frac{-4 + 1}{2} = \frac{-3}{2} = -1,5$$

Jawaban: A

21. $f(x) = \frac{2x+1}{4x-3}, x \neq \frac{3}{4}$

$$f(x) = \frac{u(x)}{v(x)} \Rightarrow f'(x) = \frac{u'(x)v(x) - v'(x)u(x)}{\{v(x)\}^2}$$

Misalkan $u(x) = 2x + 1$, maka $u'(x) = 2$

$v(x) = 4x - 3$, maka $v'(x) = 4$

Sehingga,

$$\begin{aligned} f'(x) &= \frac{2(4x-3) - 4(2x+1)}{(4x-3)^2} \\ &= \frac{8x-6-8x-4}{(4x-3)^2} \\ &= \frac{-10}{(4x-3)^2}, x \neq \frac{3}{4} \end{aligned}$$

Jawaban: A

22. $\int_0^1 3x\sqrt{3x^2+1} dx$

Misal: $u = 3x^2 + 1 \Rightarrow du = 6x dx$

$$\frac{du}{2} = 3x dx$$

Untuk $x = 0 \Rightarrow u = 1$

Untuk $x = 1 \Rightarrow u = 4$

$$\begin{aligned} \int_0^1 3x\sqrt{3x^2+1} dx &= \int_1^4 \sqrt{u} \cdot \frac{du}{2} = \frac{1}{2} \int_1^4 u^{\frac{1}{2}} du \\ &= \frac{1}{2} \left[\frac{2}{3} u^{\frac{3}{2}} \right]_1^4 = \frac{1}{3} \left[u^{\frac{3}{2}} \right]_1^4 = \frac{1}{3} \left(4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{1}{3} (8 - 1) = \frac{7}{3} \end{aligned}$$

Jawaban: C

23. Misalkan $u = \frac{1}{x}$, maka $\frac{du}{dx} = -\frac{1}{x^2}$ atau $dx = -x^2 du$

$$\begin{aligned} \int \frac{\cos\left(\frac{1}{x}\right)}{x^2} dx &= \int \frac{\cos u}{x^2} (-x^2) du \\ &= \int -\cos u du = -\sin u + C \end{aligned}$$

Kita ganti kembali fungsi u , sehingga

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^2} dx = -\sin\left(\frac{1}{x}\right) + C$$

Jawaban: C

24. Laba = harga jual – biaya produksi

$$L(x) = 5.000x - (9.000 + 1.000x + 10x^2)$$

$$L(x) = 5.000x - 9.000 - 1.000x - 10x^2$$

$$L(x) = -10x^2 + 4.000x - 9.000$$

$$L'(x) = -20x + 4.000 = 0$$

$$20x = 4.000$$

$$x = 200$$

$$\text{Laba} = -10(200)^2 + 4.000(200) - 9.000$$

$$= -10(40.000) + 800.000 - 9.000$$

$$= -400.000 + 800.000 - 9.000$$

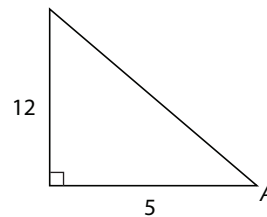
$$= 391.000$$

Jadi, laba maksimum yang dapat diperoleh perusahaan tersebut adalah Rp391.000,00.

Jawaban: C

25. A sudut lancip $\Rightarrow A$ di kuadran I (semua (+))

$$\tan A = \frac{12}{5} = \frac{de}{sa}$$

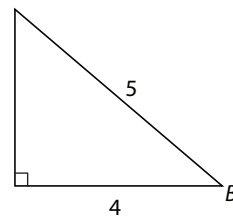


$$\begin{aligned} mi &= \sqrt{de^2 + sa^2} = \sqrt{12^2 + 5^2} \\ &= \sqrt{144 + 25} = \sqrt{169} = 13 \end{aligned}$$

Sehingga, $\sin A = \frac{12}{13}$ dan $\cos A = \frac{5}{13}$. Karena A di kuadran I, maka nilai sinus dan kosinus tetap positif.

B sudut lancip $\Rightarrow B$ di kuadran I (semua (+))

$$\sin B = \frac{4}{5} = \frac{de}{mi}$$



$$\begin{aligned} sa &= \sqrt{mi^2 - de^2} = \sqrt{5^2 - 4^2} \\ &= \sqrt{25 - 16} = \sqrt{9} = 3 \end{aligned}$$

Sehingga, $\cos B = \frac{sa}{mi} = \frac{3}{5}$. Karena B di kuadran I, maka nilai kosinus tetap positif.

$$\begin{aligned} \cos(A-B) &= \cos A \cos B + \sin A \sin B \\ &= \frac{5}{13} \cdot \frac{3}{5} + \frac{12}{13} \cdot \frac{4}{5} = \frac{15}{65} + \frac{48}{65} = \frac{63}{65} \end{aligned}$$

Jawaban: D

$$\begin{aligned} 26. \frac{\sin 7x - \sin 5x}{\cos 7x + \cos 5x} &= \frac{2 \cos\left(\frac{7x+5x}{2}\right) \sin\left(\frac{7x-5x}{2}\right)}{2 \cos\left(\frac{7x+5x}{2}\right) \cos\left(\frac{7x-5x}{2}\right)} \\ &= \frac{2 \cos\left(\frac{12x}{2}\right) \sin\left(\frac{2x}{2}\right)}{2 \cos\left(\frac{12x}{2}\right) \cos\left(\frac{2x}{2}\right)} = \frac{\sin x}{\cos x} = \tan x \end{aligned}$$

Jawaban: A

27. $3 \cos 2x + 5 \sin x + 1 = 0, \quad 0 \leq x \leq 2\pi$

$$3(1 - 2 \sin^2 x) + 5 \sin x + 1 = 0$$

$$3 - 6 \sin^2 x + 5 \sin x + 1 = 0$$

$$-6 \sin^2 x + 5 \sin x + 4 = 0$$

$$(-3 \sin x + 4)(2 \sin x + 1) = 0$$

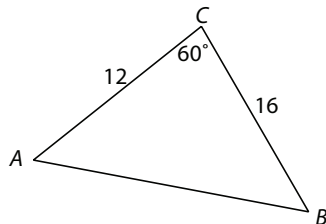
$$\sin x = \frac{4}{3} \text{ (tidak memenuhi) atau } \sin x = -\frac{1}{2}$$

$$x = \frac{7}{6}\pi, \quad x = \frac{11}{6}\pi$$

$$\text{Jadi, HP} = \left\{ \frac{7}{6}\pi, \frac{11}{6}\pi \right\}.$$

Jawaban: A

28. Diketahui $AC = 12$ m, $BC = 16$ m, dan $\angle ACB = 60^\circ$.



Berdasarkan aturan kosinus

$$AB^2 = AC^2 + BC^2 - 2 \cdot AC \cdot BC \cdot \cos 60^\circ$$

$$AB^2 = 12^2 + 16^2 - 2 \cdot 12 \cdot 16 \cdot \frac{1}{2}$$

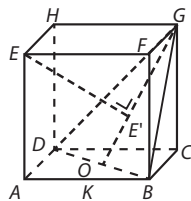
$$AB^2 = 144 + 256 - 192 = 208$$

$$AB = \sqrt{208} = 4\sqrt{13}$$

Jadi, jarak tonggak A dan B adalah $4\sqrt{13}$ m.

Jawaban: A

29. Kubus $ABCD.EFGH$ dengan panjang rusuk 8 cm.



$$EG = \text{diagonal sisi} = 8\sqrt{2} \text{ cm}$$

$$\text{Perhatikan } \triangle GOC, OC = \frac{1}{2} \times \text{diagonal sisi}$$

$$= \frac{1}{2}(8\sqrt{2}) = 4\sqrt{2}$$

$$GO = \sqrt{GC^2 + OC^2}$$

$$= \sqrt{8^2 + (4\sqrt{2})^2} = \sqrt{64 + 32} = \sqrt{96} = 4\sqrt{6}$$

$$GE' = \frac{2}{3} \times GO \text{ (E' = titik berat } \triangle BGD)$$

$$= \frac{2}{3} \times 4\sqrt{6} = \frac{8}{3}\sqrt{6}$$

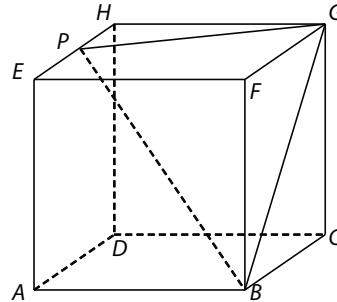
$$\text{Perhatikan } \triangle EGE', EG = \text{diagonal sisi} = 8\sqrt{2}$$

$$EE' = \sqrt{EG^2 - (GE')^2}$$

$$= \sqrt{(8\sqrt{2})^2 - \left(\frac{8}{3}\sqrt{6}\right)^2} = \sqrt{128 - \frac{128}{3}} = \sqrt{\frac{256}{3}} = \frac{16}{3}\sqrt{3}$$

Jawaban: E

30. Perhatikan gambar berikut.



$$\text{Perhatikan } \triangle BPG$$

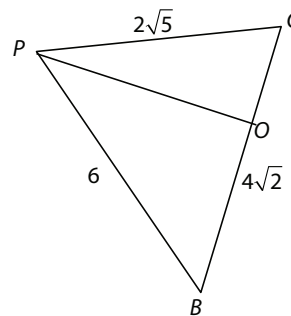
$$PG = \sqrt{PH^2 + HG^2} = \sqrt{2^2 + 4^2}$$

$$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$$

$$AP = PG = 2\sqrt{5}$$

$$BP = \sqrt{AP^2 + AB^2} = \sqrt{(2\sqrt{5})^2 + 4^2}$$

$$= \sqrt{20 + 16} = \sqrt{36} = 6$$



$$PO^2 = PO^2$$

$$PB^2 - BO^2 = PG^2 - OG^2$$

$$6^2 - (4\sqrt{2} - x)^2 = (2\sqrt{5})^2 - x^2$$

$$36 - (32 - 8x\sqrt{2} + x^2) = 20 - x^2$$

$$4 + 8x\sqrt{2} - x^2 = 20 - x^2$$

$$8x\sqrt{2} = 16$$

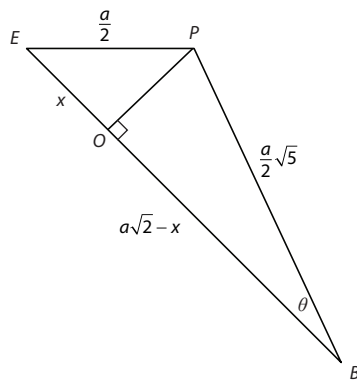
$$x = \sqrt{2}$$

$$PO = \sqrt{PG^2 - OG^2} = \sqrt{(2\sqrt{5})^2 - (\sqrt{2})^2}$$

$$= \sqrt{20 - 2} = \sqrt{18} = 3\sqrt{2}$$

Jawaban: E

$$PB = \sqrt{PF^2 + FB^2} = \sqrt{\left(\frac{a}{2}\right)^2 + a^2}$$

$$= \sqrt{\frac{a^2}{4} + a^2} = \sqrt{\frac{5a^2}{4}} = \frac{a}{2}\sqrt{5}$$


$$\begin{aligned} PO &= \sqrt{EP^2 - EO^2} = \sqrt{\left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\sqrt{2}\right)^2} \\ &= \sqrt{\frac{a^2}{4} - \frac{a^2}{8}} = \sqrt{\frac{3a^2}{8}} = \frac{a}{4}\sqrt{6} \end{aligned}$$

$$\tan \theta = \frac{PO}{OB} = \frac{\frac{3a}{4}\sqrt{2}}{\frac{a}{4}\sqrt{6}} = \frac{3\sqrt{2}}{\sqrt{6}} = \sqrt{3}$$

32. Vektor $\vec{a} = 4\vec{i} - 2\vec{j} + 2\vec{k}$
Vektor $\vec{b} = 2\vec{i} - 6\vec{j} + 4\vec{k}$

Proyeksi vektor orthogonal vektor $\vec{a}$ pada vektor $\vec{b}$ adalah: $\vec{a}_b = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \cdot \vec{b}$

$$\begin{aligned}\vec{a}_6 &= \frac{4 \cdot 2 + (-2)(-6) + 2 \cdot 4}{\left(\sqrt{2^2 + (-6)^2 + 4^2}\right)^2} \cdot (2\vec{i} - 6\vec{j} + 4\vec{k}) \\ &= \frac{28}{56}(2\vec{i} - 6\vec{j} + 4\vec{k}) = \frac{1}{2}(2\vec{i} - 6\vec{j} + 4\vec{k}) = \vec{i} - 3\vec{j} + 2\vec{k}\end{aligned}$$

$$\begin{aligned} 33. \quad & x^2 + y^2 - 6x + ay + 16 = 0 \\ & x^2 - 6x + 9 + y^2 + ay + \frac{a^2}{4} = \frac{a^2}{4} - 7 \\ & (x-3)^2 + \left(y + \frac{a}{2}\right)^2 = \frac{a^2}{4} - 7 \end{aligned}$$
$$r = \sqrt{\frac{a^2}{4} - 7}.$$
$$3 = \sqrt{\frac{a^2}{4} - 7}$$

$$9 = \frac{a^2}{4} - 7$$

$$\begin{aligned}\frac{a^2}{4} &= 16 \\ a^2 &= 64 \\ a &= \pm 8\end{aligned}$$

Jawaban: B

dengan pusat $\left(-\frac{1}{2}A, -\frac{1}{2}B\right)$.

melalui $(5, -1) \Rightarrow (5)^2 + (-1)^2 - 4(5) - 6(-1) + C = 0$

$$25 + 1 - 20 + 6 + C = 0$$

$$12 + C = 0$$

$$C = -12$$

Jawaban: A

35. Pencermian terhadap sumbu-X, $T_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Dilatasi dengan pusat O dan faktor skala 3, $T_2 = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

Pencermian terhadap sumbu-X dilanjutkan dilatasi [O, 3] adalah:

$$T_2 \circ T_1 = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & -3 \end{pmatrix}$$

Sehingga $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-9} \begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{1}{3}x' \\ -\frac{1}{3}y' \end{pmatrix}$$

Diperoleh $x = \frac{1}{3}x'$ dan $y = -\frac{1}{3}y'$

Bayangan $y = x^2 - 3x + 2$ adalah

$$-\frac{1}{3}y' = \left(\frac{1}{3}x'\right)^2 - 3\left(\frac{1}{3}x'\right) + 2 \quad (\text{dikali } 3)$$

$$-y = \frac{1}{3}x^2 - 3x + 6$$

$$-3y = x^2 - 9x + 18$$

$$0 = 3y + x^2 - 9x + 18$$

$$3y + x^2 - 9x + 18 = 0$$

Jawaban: A

36. Diagram pada soal dapat diubah ke dalam bentuk tabel berikut.

Data	x_i	f	$f_i \cdot x_i$
41-45	43	5	215
46-50	48	11	528
51-55	53	9	477
56-60	58	4	232
61-65	63	1	63
Σ		30	1.515

$$\bar{x} = \frac{\sum f_i \cdot x_i}{\sum f_i}$$

$$= \frac{1.515}{30} = 50,5$$

Jadi, nilai rataannya adalah 50,5.

Jawaban: D

37. $n = \sum f_i = 24$

Kuartil atas = Q_3

Kelas Q_3 berada pada data ke-

$$\frac{3}{4} \times n = \frac{3}{4} \times 24 = 18 \quad (\text{kelas ke-4})$$

$$T_b = 56 - 0,5 = 55,5$$

Panjang kelas, $I = 3$

$$Q_3 = T_b + \left(\frac{\frac{3}{4}n - F_k}{f_{Q_3}} \right) I$$

$$= 55,5 + \left(\frac{18 - 13}{7} \right) 3$$

$$= 55,5 + \frac{5}{7} \times 3$$

$$= 55,5 + \frac{15}{7}$$

$$= 55,5 + 2,14 = 57,64$$

Jawaban: D

38. 5 soal harus dikerjakan, 4 soal sisanya harus dipilih dari 5 soal yang masih tersedia.

$$C_{(5,4)} = \frac{5!}{(5-4)! 4!} = \frac{5 \times 4!}{1 \times 4!} = 5$$

Banyaknya pilihan = 5 cara

Jawaban: B

39. Banyak soal bernomor genap ada 5, sehingga 5 soal harus dikerjakan, 2 soal sisanya harus dipilih dari 5 soal yang masih tersedia.

$${}_2C_5 = \frac{5!}{(5-2)! 2!} = \frac{5 \cdot 4 \cdot 3!}{2 \cdot 1 \cdot 3!} = 5 \cdot 2 = 10$$

Jadi, banyak cara peserta ujian memilih soal ujian yang harus dikerjakan adalah 10.

Jawaban: D

40. Semua + keduanya = satu + satu + tidak

$$100 + 25 = 50 + 45 + x \Leftrightarrow x = 30$$

Yang tidak mengikuti kuliah sastra maupun sejarah ada 30.

$$\text{Peluang terpilihnya} = \frac{30}{100} = 0,3$$

Jawaban: E

PEMBAHASAN DAN KUNCI JAWABAN PEMANTAPAN PAKET 1

MATEMATIKA IPA

$$1. \frac{72a^2b^{-5}c^7}{-4b^2a^{-3}c^9} = \frac{18a^2a^3c^7}{-b^2b^5c^9} = \frac{18a^5}{-b^7c^2}$$

Jawaban: B

$$\begin{aligned} 2. & (\sqrt{2} + \sqrt{3} + 2 + \sqrt{5})(-\sqrt{2} + \sqrt{3} + 2 - \sqrt{5})(\sqrt{10} + 2\sqrt{3}) \\ &= ((\sqrt{2} + \sqrt{3}) + (2 + \sqrt{5}))((\sqrt{2} + \sqrt{3}) - (\sqrt{2} + \sqrt{5}))(\sqrt{10} + 2\sqrt{3}) \\ &= ((2 + \sqrt{3})^2 - (\sqrt{2} + \sqrt{5})^2)(\sqrt{10} + 2\sqrt{3}) \\ &= ((7 + 4\sqrt{3}) - (7 + 2\sqrt{10}))(\sqrt{10} + 2\sqrt{3}) \\ &= (4\sqrt{3} - 2\sqrt{10})(\sqrt{10} + 2\sqrt{3}) \\ &= -2(\sqrt{10} - 2\sqrt{3})(\sqrt{10} + 2\sqrt{3}) = -2(10 - 12) = 4 \end{aligned}$$

Jawaban: E

$$\begin{aligned} 3. & x+y \log 2 = a \Leftrightarrow {}^2\log(x+y) = \frac{1}{a} \\ & x-y \log 8 = b \Leftrightarrow {}^8\log(x-y) = \frac{1}{b} \Leftrightarrow {}^{2^3}\log(x-y) = \frac{1}{b} \\ & \Leftrightarrow \frac{1}{3} {}^2\log(x-y) = \frac{1}{b} \Leftrightarrow {}^2\log(x-y) = \frac{3}{b} \\ & {}^4\log(x^2 - y^2) = {}^2\log(x+y)(x-y) \\ &= \frac{1}{2} \{ {}^2\log(x+y) + {}^2\log(x-y) \} \\ &= \frac{1}{2} \left\{ \frac{1}{a} + \frac{3}{b} \right\} = \frac{1}{2} \left\{ \frac{3a+b}{ab} \right\} = \frac{3a+b}{2ab} \end{aligned}$$

Jawaban: D

$$4. 2 \log x \leq \log(x+3) + \log 4$$

Syarat pertidaksamaan

$$\Leftrightarrow \log x^2 \leq \log 4(x+3) \Leftrightarrow x^2 \leq 4x + 12$$

$$\Leftrightarrow x^2 - 4x - 12 \leq 0 \Leftrightarrow (x-6)(x+2) \leq 0$$

$$\Leftrightarrow -2 \leq x \leq 6 \dots (1)$$

Syarat numerus

$$x+3 > 0 \Leftrightarrow x > -3 \dots (2)$$

$$x > 0 \dots (3)$$

Dari (1), (2), dan (3) diperoleh irisan $\{x \mid 0 < x \leq 6\}$.

Jawaban: C

$$5. \text{ Diketahui } g(x) = x + 1.$$

Cara I

$$(f \circ g)(x) = x^2 + 3x + 1$$

$$\Leftrightarrow f(g(x)) = x^2 + 3x + 1$$

$$\Leftrightarrow f(x+1) = x^2 + 3x + 1$$

$$\text{Misalkan } y = x + 1 \Leftrightarrow x = y - 1$$

$$\Leftrightarrow f(y) = (y-1)^2 + 3(y-1) + 1$$

$$f(y) = y^2 - 2 - 2y + 1 + 3y - 3 + 1 \Leftrightarrow f(y) = y^2 + y - 1$$

$$\text{Sehingga } f(x) = x^2 + x - 1$$

Cara II

Pilih nilai x sembarang, misalnya $x = 0$ (untuk memudahkan)

$$(f \circ g)(x) = x^2 + 3x + 1$$

$$\Leftrightarrow f(g(x)) = x^2 + 3x + 1 \Leftrightarrow f(g(0)) = 0^2 + 3(0) + 1$$

$$\Leftrightarrow f(g(0)) = 1$$

Karena $g(0) = 1$, maka diperoleh $f(1) = 1$.

Kita cek setiap jawaban, mana yang memenuhi $f(1) = 1$

$$A. x^2 + 5x + 5 = 1^2 + 5(1) + 5 = 11 \neq 1$$

$$B. x^2 + x - 1 = 1^2 + 1 - 1 = 1 \quad (\text{memenuhi})$$

$$C. x^2 + 4x + 3 = 1^2 + 4(1) + 3 = 8 \neq 1$$

$$D. x^2 + 6x + 1 = 1^2 + 6(1) + 1 = 8 \neq 1$$

$$E. x^2 + 3x - 1 = 1^2 + 3(1) - 1 = 3 \neq 1$$

Jawaban: B

$$\begin{aligned} 6. & (g \circ f)(x) = g(f(x)) = g\left(\frac{x+3}{x-1}\right) = \frac{x+3}{x-1} + 5 \\ &= \frac{x+3+5(x-1)}{x-1} = \frac{6x-2}{x-1}, x \neq 1 \end{aligned}$$

Ingat rumus invers:

$$\text{Jika } f(x) = \frac{ax+b}{cx+d}, \text{ maka } f^{-1}(x) = \frac{-dx+b}{cx-a}$$

$$(g \circ f)(x) = \frac{6x-2}{x-1} \Rightarrow (g \circ f)^{-1}(x) = \frac{x-2}{x-6}, x \neq 6$$

Jawaban: B

$$7. x_1 \text{ dan } x_2 \text{ akar-akar dari } 3x^2 - (a-1)x - 1 = 0, \text{ sehingga}$$

$$x_1 + x_2 = \frac{a-1}{3} \dots (1)$$

$$x_1 x_2 = -\frac{1}{3} \dots (2)$$

Persamaan $x^2 - (2b+1)x + b = 0$ akar-akarnya $\frac{1}{x_1}$ dan

$\frac{1}{x_2}$, sehingga

$$\frac{1}{x_1} + \frac{1}{x_2} = 2b + 1 \dots (3)$$

$$\frac{1}{x_1} \cdot \frac{1}{x_2} = \frac{1}{x_1 x_2} = b \dots (4)$$

Substitusikan (2) ke (4), diperoleh

$$\frac{1}{-\frac{1}{3}} = b \Leftrightarrow b = -3$$

Substitusikan $b = -3$ ke (3), diperoleh

$$\frac{1}{x_1} + \frac{1}{x_2} = 2(-3) + 1 \Leftrightarrow \frac{x_1 + x_2}{x_1 x_2} = -5$$

$$\Leftrightarrow \frac{a-1}{-\frac{1}{3}} = -5 \Leftrightarrow \frac{a-1}{3} = \frac{5}{3} \Leftrightarrow a = 6$$

Nilai $2a + b = 2(6) - 3 = 9$

Jawaban: C

8. Misalkan akar-akar dari $4x^2 - 2mx + 2m - 3 = 0$ adalah x_1 dan x_2 , sehingga,

Kedua akar real berbeda $\Rightarrow D > 0$

Kedua akar positif $\Rightarrow x_1 + x_2 > 0$

$$D = b^2 - 4ac > 0$$

$$D = (-2m)^2 - 4(4)(2m - 3) > 0 \Leftrightarrow 4m^2 - 32m + 48 > 0$$

$$\Leftrightarrow m^2 - 8m + 12 > 0 \Leftrightarrow (m - 6)(m - 2) > 0$$

$$\Leftrightarrow m < 2 \text{ atau } m > 6 \quad \dots (1)$$

$$x_1 + x_2 > 0 \Leftrightarrow \frac{m}{2} > 0 \Leftrightarrow m > 0 \quad \dots (2)$$

Dari (1) dan (2) diperoleh irisan $0 < m < 2$ atau $m > 6$

Jawaban: C

9. (1) $\sim(p \wedge q) \equiv \sim p \vee \sim q$ (pernyataan benar)
 (2) $\sim(p \Rightarrow q) \equiv \sim(\sim p \vee q) \equiv p \wedge \sim q$ (pernyataan benar)
 (3) $\sim(p \vee q) \equiv \sim p \wedge \sim q$ (pernyataan benar)
 (4) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ hukum distributif (pernyataan benar)

Jawaban: E

10. Diketahui parabola $y = x^2 - px + 3$ dan garis $y = 2x - 1$.

Substitusikan persamaan garis ke parabola, diperoleh

$$x^2 - px + 3 = 2x - 1 \Leftrightarrow x^2 + (-p - 2)x + 4 = 0$$

Syarat memotong di dua titik $\Rightarrow D > 0$

$$D = b^2 - 4ac > 0$$

$$(p - 2)^2 - 4(1)(4) > 0$$

$$\Leftrightarrow p^2 + 4p - 12 > 0 \Leftrightarrow (p + 6)(p - 2) > 0$$

$$\Leftrightarrow p < -6 \text{ atau } p > 2$$

Jawaban: A

11. Persoalan tersebut dapat ditulis dalam bentuk sistem persamaan linear

$$\begin{array}{rcl} 6K + 4L = 74.000 & \left| \begin{array}{l} \times 1 \\ \times 2 \end{array} \right| & \begin{array}{l} 6K + 4L = 74.000 \\ 10K + 4L = 110.000 \\ \hline -4K = -36.000 \\ K = 9.000 \end{array} \end{array}$$

$$K = 9.000 \Rightarrow 5(9.000) + 2L = 55.000$$

$$2L = 55.000 - 45.000 = 10.000$$

$$L = 5.000$$

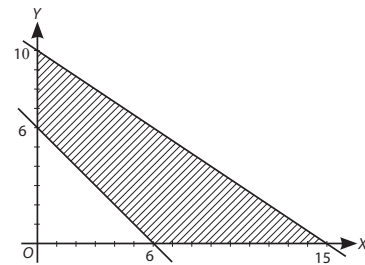
Dengan demikian,

$$\begin{aligned} 5K + 5L &= 5(9.000) + 5(5.000) \\ &= 45.000 + 25.000 \\ &= 70.000 \end{aligned}$$

Jadi, upah yang diperoleh Pak Dodo adalah Rp70.000,00

Jawaban: D

12. Perhatikan gambar berikut.



Kita peroleh titik-titik pojok (6, 0), (0, 6), (15, 0), dan (0, 10).

$$F = 150x + 100y$$

$$(6, 0) \Rightarrow F = 150(6) + 100(0) = 900$$

$$(0, 6) \Rightarrow F = 150(0) + 100(6) = 600 \text{ (minimum)}$$

$$(15, 0) \Rightarrow F = 150(15) + 100(0) = 2.250 \text{ (maksimum)}$$

$$(0, 10) \Rightarrow F = 150(0) + 100(10) = 1.000$$

$$F_{\text{maks}} : F_{\text{min}} = 2.250 : 600 = 15 : 4$$

Jawaban: D

$$13. \begin{pmatrix} 4 & 1 \\ 3 & a \end{pmatrix} \begin{pmatrix} -1 & a \\ 2a+b & 7 \end{pmatrix} = \begin{pmatrix} 1 & 15 \\ 7 & 20 \end{pmatrix}$$

$$\begin{pmatrix} 4(-1) + 2a+b & 4a+7 \\ 3(-1) + a(2a+b) & 10a \end{pmatrix} = \begin{pmatrix} 1 & 15 \\ 7 & 20 \end{pmatrix}$$

$$\text{Diperoleh: } 10a = 20 \Rightarrow a = 2$$

$$-4 + 2a + b = 1$$

$$-4 + 2(2) + b = 1$$

$$b = 1$$

Jawaban: A

$$14. P = \begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix} \Leftrightarrow P^{-1} = \frac{-1}{2} \begin{pmatrix} 6 & -4 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 1 & -\frac{1}{2} \end{pmatrix}$$

$$PX = P^{-1} \Leftrightarrow X = P^{-1}P^{-1}$$

$$\Leftrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 1 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} -3 & 2 \\ 1 & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 11 & -7 \\ -\frac{7}{2} & \frac{9}{4} \end{pmatrix}$$

$$\text{Nilai } a+b+c+d = 11 - 7 - \frac{7}{2} + \frac{9}{4} = \frac{11}{4}$$

Jawaban: A

$$15. U_6 + U_9 + U_{12} + U_{15} = 20$$

$$\Leftrightarrow a + 5b + a + 8b + a + 11b + a + 14b = 20$$

$$\Leftrightarrow 4a + 38b = 20$$

$$\Leftrightarrow 2a + 19b = 10$$

$$S_n = \frac{n}{2}(2a + (n-1)b)$$

$$S_{20} = \frac{20}{2}(2a + 19b) = 10(10) = 100$$

Jawaban: C

16. $a, 8, c, d$ merupakan barisan aritmetika, sehingga

$$2(8) = a + c \Leftrightarrow a + c = 16 \quad \dots (1)$$

$$2c = d + 8 \Leftrightarrow 2c - d = 8 \quad \dots (2)$$

$a, 8, d$ merupakan barisan geometri, sehingga

$$8^2 = ad \Leftrightarrow ad = 64 \quad \dots (3)$$

$$(1) \quad a + c = 16 \quad \times 2 \quad \left| \begin{array}{l} 2a + 2c = 32 \\ 2c - d = 8 \end{array} \right|$$

$$(2) \quad 2c - d = 8 \quad \times 1 \quad \left| \begin{array}{l} 2c - d = 8 \\ a + d = 24 \end{array} \right|$$

$$a + d = 24 \Leftrightarrow d = 24 - 2a \quad \dots (4)$$

Substitusikan (4) ke (3), diperoleh

$$a(24 - 2a) = 64 \Leftrightarrow 2a^2 - 24a + 64 = 0$$

Kedua ruas dibagi 2, diperoleh

$$\Leftrightarrow a^2 - 12a + 32 = 0 \Leftrightarrow (a - 8)(a - 4) = 0$$

$$\Leftrightarrow a = 8 \text{ atau } a = 4$$

Untuk $a = 8 \Rightarrow d = 24 - 2(8) = 8$, sehingga nilai

$$a + c + d = 16 + 8 = 24$$

Untuk $a = 4 \Rightarrow d = 24 - 2(4) = 16$, sehingga nilai

$$a + c + d = 16 + 16 = 32$$

Jawaban: C

$$17. U_4 = ar^3 = 48$$

$$6r^3 = 48$$

$$r^3 = 8$$

$$r = 2$$

$$S_6 = \frac{a(r^6 - 1)}{r - 1} = \frac{6(2^6 - 1)}{2 - 1} = \frac{6(64 - 1)}{1} = 378$$

Jawaban: C

$$18. P(x) \text{ dibagi } (x - 1) \text{ sisa } 11 \Rightarrow P(1) = 11$$

$$\Leftrightarrow 2(1)^4 + a(1)^3 - 3(1)^2 + 5(1) + b = 11$$

$$\Leftrightarrow a + b = 7 \quad \dots (1)$$

$$P(x) \text{ dibagi } (x + 1) \text{ sisa } -1 \Rightarrow P(-1) = -1$$

$$\Leftrightarrow 2(-1)^4 + a(-1)^3 - 3(-1)^2 + 5(-1) + b = -1$$

$$\Leftrightarrow -a + b = 5 \quad \dots (2)$$

Dari (1) dan (2) diperoleh $a = 1$ dan $b = 6$

$$\text{Nilai } 2a + b = 2(1) + 6 = 8$$

Jawaban: C

$$19. \lim_{x \rightarrow 3} \frac{\sqrt{x+4} - \sqrt{2x+1}}{x-3}$$

Karena bentuknya $\frac{0}{0}$, kita gunakan aturan L'Hopital

$$\Leftrightarrow = \lim_{x \rightarrow 3} \frac{(x+4)^{\frac{1}{2}} - (2x+1)^{\frac{1}{2}}}{x-3}$$

$$\Leftrightarrow = \lim_{x \rightarrow 3} \frac{\frac{1}{2}(x+4)^{-\frac{1}{2}} - \frac{1}{2}(2x+1)^{-\frac{1}{2}}}{1}$$

$$\Leftrightarrow = \lim_{x \rightarrow 3} \frac{1}{2\sqrt{x+4}} - \frac{1}{\sqrt{2x+1}}$$

$$\Leftrightarrow = \frac{1}{2\sqrt{3+4}} - \frac{1}{\sqrt{2(3)+1}} = \frac{1}{2\sqrt{7}} - \frac{1}{\sqrt{7}}$$

$$\Leftrightarrow = \frac{\sqrt{7}}{14} - \frac{\sqrt{7}}{7} = -\frac{\sqrt{7}}{14}$$

Jawaban: B

$$20. \lim_{x \rightarrow \infty} (\sqrt{4x^2 + 3x + 1} - 2x)$$

$$= \lim_{x \rightarrow \infty} (\sqrt{4x^2 + 3x + 1} - \sqrt{4x^2})$$

$$= \frac{3-0}{2\sqrt{4}} = \frac{3}{2(2)} = \frac{3}{4}$$

Jawaban: B

21. Diketahui kurva $f(x) = x^3 + ax^2 + bx + c$ turun pada interval $-5 < x < 3$.

$$f'(x) = 3x^2 + 2ax + b$$

Titik stasioner $f'(x) = 0$

$$3x^2 + 2ax + b = 0$$

$$x^2 + \frac{2a}{3}x + \frac{b}{3} = 0 \quad \dots (1)$$

$f(x)$ turun pada interval $-5 < x < 3$, artinya titik stasionernya adalah $x = -5$ dan $x = 3$.

Sehingga,

$$(x+5)(x-3) = 0$$

$$x^2 + 2x - 15 = 0 \quad \dots (2)$$

Dari persamaan (1) dan (2), diperoleh:

$$\frac{2a}{3} = 2 \Leftrightarrow a = 3$$

$$\frac{b}{3} = -15 \Leftrightarrow b = -45$$

$$f(x) = x^3 + 3x^2 - 45x + c$$

$f(x)$ melalui $(2, -13)$, sehingga

$$-13 = (2)^3 + 3(2)^2 - 45(2) + c$$

$$-13 = 8 + 12 - 90 + c$$

$$-13 = -70 + c$$

$$c = -13 + 70 = 57$$

Jawaban: E

22. Misalkan: $u = x^3 + 8 \Rightarrow du = 3x^2 dx$

$$3 du = 9x^2 dx$$

$$\int \frac{9x^2}{\sqrt{x^3+8}} dx = \int \frac{1}{\sqrt{u}} 3 du = 3 \int u^{-\frac{1}{2}} du = \frac{3}{\frac{1}{2}} u^{\frac{1}{2}} + C$$

$$= 6u^{\frac{1}{2}} + C = 6\sqrt{x^3+8} + C$$

Jawaban: D

23. $\int_{-1}^2 (x^2 + x - 2) dx = \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x \right]_{-1}^2$

$$= \left[\frac{1}{3}(2)^3 + \frac{1}{2}(2)^2 - 2(2) \right] - \left[\frac{1}{3}(-1)^3 + \frac{1}{2}(-1)^2 - 2(-1) \right]$$

$$= \left(\frac{8}{3} + 2 - 4 \right) - \left(-\frac{1}{3} + \frac{1}{2} + 2 \right)$$

$$= \frac{8}{3} - 2 + \frac{1}{3} - \frac{1}{2} - 2 = -1\frac{1}{2}$$

Jawaban: C

24. Biaya per jam $\left(4x - 800 + \frac{120}{x} \right)$ ratus ribu rupiah

Biaya x jam $= \left(4x - 800 + \frac{120}{x} \right) x$ ratus ribu rupiah

$B(x) = (4x^2 - 800x + 120)$ ratus ribu rupiah

Agar $B(x)$ minimum, $B'(x) = 0$

$$8x - 800 = 0$$

$$x = 100$$

Jadi, agar biaya minimum, produk tersebut harus diselesaikan dalam waktu 100 jam.

Jawaban: C

25. $\triangle ABC$ siku-siku di $C \Rightarrow C = 90^\circ$, sehingga

$$A + B + C = 180^\circ \Leftrightarrow A + B = 90^\circ \Leftrightarrow B = 90^\circ - A$$

$$2 \tan A = \sin B \Leftrightarrow 2 \frac{\sin A}{\cos A} = \sin(90^\circ - A)$$

$$\Leftrightarrow 2 \frac{\sin A}{\cos A} = \cos A \Leftrightarrow 2 \sin A = \cos^2 A$$

$$\Leftrightarrow 2 \sin A = 1 - \sin^2 A \Leftrightarrow \sin^2 A + 2 \sin A - 1 = 0$$

$$\Leftrightarrow \sin A = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)} = -1 \pm \sqrt{2}$$

$$\Leftrightarrow \sin A = -1 + \sqrt{2} \text{ atau } \sin A = -1 - \sqrt{2}$$

Jawaban: C

26. $\frac{\sin 78^\circ - \sin 12^\circ}{\cos 168^\circ - \cos 102^\circ} = \frac{2 \cos \left(\frac{78^\circ + 12^\circ}{2} \right) \sin \left(\frac{78^\circ - 12^\circ}{2} \right)}{-2 \sin \left(\frac{168^\circ + 102^\circ}{2} \right) \sin \left(\frac{168^\circ - 102^\circ}{2} \right)}$

$$= \frac{2 \cos 45^\circ \sin 33^\circ}{-2 \sin 135^\circ \sin 33^\circ} = -\frac{\cos 45^\circ}{\sin 135^\circ}$$

$$= -\frac{\frac{1}{2}\sqrt{2}}{\frac{1}{2}\sqrt{2}} = -1$$

Jawaban: A

27. $3 \cos^2 2x + 4 \cos 2x - 4 = 0$

$$\Leftrightarrow (3 \cos 2x - 2)(\cos 2x + 2) = 0$$

$$\Leftrightarrow \cos 2x = \frac{2}{3} \text{ atau } \cos 2x = -2$$

$$\bullet \cos 2x = \frac{2}{3} \Rightarrow 2 \cos^2 x - 1 = \frac{2}{3}$$

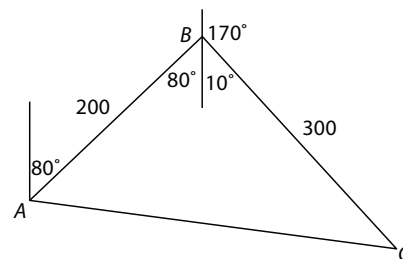
$$\Leftrightarrow \cos^2 x = \frac{5}{6} \Leftrightarrow \cos x = \pm \sqrt{\frac{5}{6}} = \pm \frac{\sqrt{30}}{6}$$

$$\Leftrightarrow \cos x = \frac{\sqrt{30}}{6} \text{ atau } \cos x = -\frac{\sqrt{30}}{6}$$

$$\bullet \cos 2x = -2, \text{ tidak ada } x \text{ yang memenuhi}$$

Jawaban: D

28. Kapal berlayar dari A ke B sejauh 200 km dengan arah 080° . Kemudian dari B ke C sejauh 300 km dengan arah 170° .



$$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos 90^\circ$$

$$AC^2 = 200^2 + 300^2 - 2 \cdot 200 \cdot 300 \cdot 0$$

$$AC^2 = 40.000 + 90.000 - 0$$

$$AC^2 = 130.000$$

$$AC = \sqrt{130.000} = 100\sqrt{13}$$

Jadi, jarak antara pelabuhan A dan C adalah $100\sqrt{13}$ km.

Jawaban: D

29. Perhatikan gambar berikut.

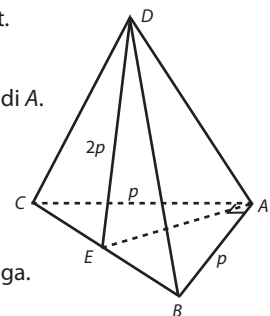
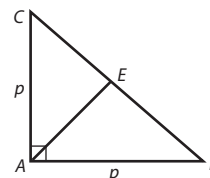
Perhatikan $\triangle ABC$ siku-siku di A .

$$BC = \sqrt{AB^2 + AC^2}$$

$$= \sqrt{p^2 + p^2}$$

$$= p\sqrt{2}$$

Gunakan rumus luas segitiga.



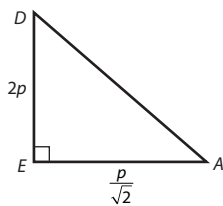
$$AB \cdot AC = BC \cdot AE$$

$$p \cdot p = p\sqrt{2} \cdot AE$$

$$AE = \frac{p}{\sqrt{2}}$$

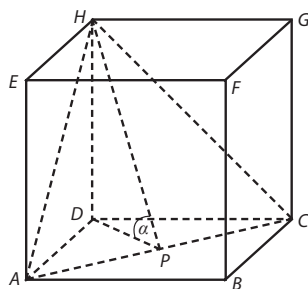
Perhatikan $\triangle ADE$. Karena $DE \perp BC$, maka $DE \perp AE$.

$$\begin{aligned} AD &= \sqrt{AE^2 + DE^2} \\ &= \sqrt{\left(\frac{p}{\sqrt{2}}\right)^2 + (2p)^2} \\ &= \sqrt{\frac{p^2}{2} + 4p^2} = \sqrt{\frac{9}{2}p^2} \\ &= \frac{3p}{\sqrt{2}} = \frac{3p}{2}\sqrt{2} \end{aligned}$$



Jawaban: B

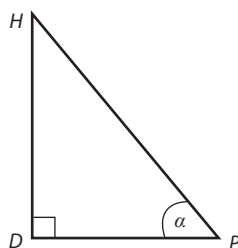
30.



Misalkan panjang rusuk kubus = a .

$$DH = a$$

$$DP = \frac{1}{2}BD = \frac{1}{2}a\sqrt{2}$$



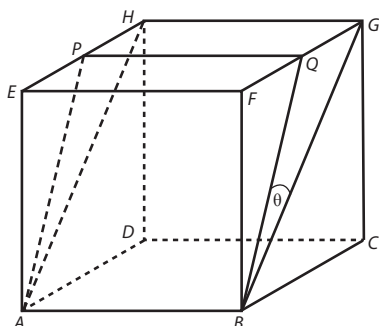
$$\begin{aligned} PH &= \sqrt{DP^2 + DH^2} \\ &= \sqrt{\left(\frac{1}{2}a\sqrt{2}\right)^2 + a^2} \\ &= \sqrt{\frac{3}{2}a^2} = \frac{1}{2}a\sqrt{6} \end{aligned}$$

Sehingga,

$$\cos \alpha = \frac{DP}{PH} = \frac{\frac{1}{2}a\sqrt{2}}{\frac{1}{2}a\sqrt{6}} = \frac{\sqrt{2}}{\sqrt{6}} = \frac{\sqrt{12}}{6} = \frac{2\sqrt{3}}{6} = \frac{1}{3}\sqrt{3}$$

Jawaban: B

31. Perhatikan gambar berikut ini.



Misalkan rusuk kubus = $2a$.

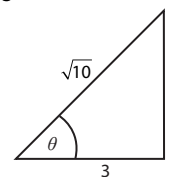
$$BQ = \sqrt{BF^2 + FQ^2} = \sqrt{(2a)^2 + a^2} = a\sqrt{5}$$

$$BG = 2a\sqrt{2} \quad (\text{diagonal sisi})$$

$$GQ = a$$

$$\begin{aligned} \cos \theta &= \frac{BQ^2 + BG^2 - GQ^2}{2 \cdot BQ \cdot BG} \\ &= \frac{(a\sqrt{5})^2 + (2a\sqrt{2})^2 - a^2}{2(a\sqrt{5})(2a\sqrt{2})} = \frac{3}{\sqrt{10}} \end{aligned}$$

$$\tan \theta = \frac{1}{3}$$



Jawaban: D

$$32. (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) = \vec{u} \cdot \vec{u} + \vec{v} \cdot \vec{v}$$

$$\Leftrightarrow \vec{u} \cdot \vec{u} + 2\vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v} = \vec{u} \cdot \vec{u} + \vec{v} \cdot \vec{v}$$

$$\Leftrightarrow 2\vec{u} \cdot \vec{v} = 0 \Leftrightarrow \vec{u} \cdot \vec{v} = 0$$

Jadi, $\vec{u}$ tegak lurus $\vec{v}$. Sehingga segitiga ABC adalah segitiga siku-siku. Pada segitiga siku-siku, sudut terbesarnya adalah 90° .

Jawaban: C

33. Pusat $(2, 4)$ dan menyinggung garis $y = 2$, maka $r = 2$
Persamaan lingkaran:

$$(x - 2)^2 + (y - 4)^2 = 2^2$$

$$x^2 - 4x + 4 + y^2 - 8y + 16 = 4$$

$$x^2 + y^2 - 4x - 8y + 16 = 0$$

Jawaban: B

34. Persamaan garis singgung lingkaran

$$x^2 + y^2 + Ax + By + C = 0 \text{ di titik } (x_1, y_1) \text{ adalah}$$

$$x_1 \cdot x + y_1 \cdot y + \frac{1}{2}A(x + x_1) + \frac{1}{2}B(y + y_1) + C = 0$$

Sehingga persamaan garis singgungnya adalah

$$2x - y + \frac{1}{2}(-2)(x + 2) + \frac{1}{2}(8)(y - 1) + 10 = 0$$

$$2x - y - x - 2 + 4y - 4 + 10 = 0$$

$$x + 3y + 4 = 0$$

Jawaban: E

$$35. \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2 & -4 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & -4 \\ 4 & 5 \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{5}{6} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\Leftrightarrow x = \frac{5}{6}x' + \frac{2}{3}y' \text{ dan } y = -\frac{2}{3}x' - \frac{1}{3}y'$$

Substitusikan ke persamaan $y = 6x - 3$, diperoleh

$$-\frac{2}{3}x' - \frac{1}{3}y' = 6\left(\frac{5}{6}x' + \frac{2}{3}y'\right) - 3$$

$$\Leftrightarrow -\frac{2}{3}x' - \frac{1}{3}y' = 5x' + 4y' - 3$$

Kedua ruas dikali 3, diperoleh

$$-2x' - y' = 15x' + 12y' - 9$$

$$\Leftrightarrow 17x' + 13y' = 9$$

Jadi, persamaan garis bayangannya adalah

$$17x + 13y = 9.$$

Jawaban: B

36. Dari diagram pada soal dapat dilihat kelas modus adalah interval (45,5–50,5) karena frekuensinya paling banyak.

Dengan demikian,

$$T_b = 45,5, s_1 = 20 - 17 = 3, s_2 = 20 - 13 = 7, C = 5$$

$$Mo = T_b + \frac{s_1}{s_1 + s_2} \cdot C$$

$$= 45,5 + \frac{3}{3+7} \cdot 5$$

$$= 45,5 + \frac{3}{10} \cdot 5 = 45,5 + 1,5 = 47$$

Jawaban: C

37. Letak kuartil atas $\Rightarrow \frac{3}{4}n = \frac{3}{4}(40) = 30$ (kelas ke-4)

$$T_b = 60,5 \quad \Sigma f = 29 \quad f_{Q_3} = 10 \quad c = 10$$

$$Q_3 = T_b + \frac{\frac{3}{4}n - \Sigma f}{f_{Q_3}} \cdot c = 60,5 + \frac{30 - 29}{10} (10) = 61,5$$

Jawaban: B

38.

L	P	P	P	L
2	3	2	1	1

Perhatikan tabel di atas. Untuk mengisi posisi kesatu, karena ada 2 orang laki-laki, maka ada 2 cara untuk mengisinya. Untuk posisi kedua, karena ada 3

perempuan, maka ada 3 cara untuk mengisinya. Posisi ke-3 dapat diisi oleh 2 perempuan karena 1 orang perempuan telah mengisi posisi 1. Untuk posisi ke-4, hanya sisa satu perempuan yang dapat mengisinya. Posisi ke-5 dapat diisi oleh 1 laki-laki karena laki-laki yang lain telah mengisi posisi pertama.

Jadi, banyak formasi yang dapat dibentuk

$$= 2 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 12$$

Jawaban: C

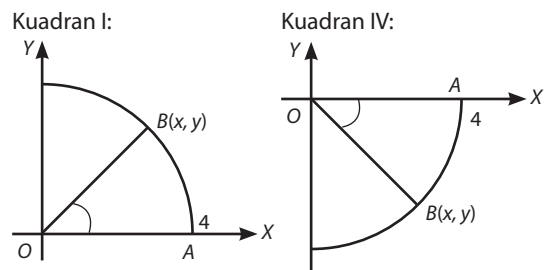
39. Ada 10 soal, nomor 1 sampai dengan nomor 5 harus dikerjakan berarti tersisa 5 soal yang bisa dipilih.

Harus mengerjakan 9 soal, berarti harus memilih 4 soal dari 5 soal yang bisa dipilih.

$$\text{Banyaknya cara} = C_4^5 = 5$$

Jawaban: B

40. Titik-titik A, O, dan B akan membentuk sudut lancip jika berada di kuadran I dan IV.



Di kuadran I dan IV, titik B membentuk $\frac{1}{2}$ lingkaran.

$$\text{Keliling } \frac{1}{2} \text{ lingkaran} = \frac{1}{2} \cdot 2\pi \cdot r = \frac{1}{2} \cdot 2\pi \cdot 4 = 4\pi$$

$$\text{Peluang } \angle AOB \text{ lancip} = \frac{4\pi}{2\pi r} = \frac{4\pi}{2\pi(4)} = \frac{1}{2}$$

Jawaban: C

PEMBAHASAN DAN KUNCI JAWABAN SOAL PEMANTAPAN PAKET 2

**MATEMATIKA
IPA**

$$\begin{aligned} 1. & \frac{(a+b)^{-1}(a^{-2}-b^{-2})}{(a^{-1}+b^{-1})(ab^{-1}-a^{-1}b)} \\ &= \left(\frac{1}{a+b}\right)\left(\frac{1}{a^2}-\frac{1}{b^2}\right) = \left(\frac{1}{a+b}\right)\left(\frac{b^2-a^2}{a^2b^2}\right) = \frac{b^2-a^2}{(a+b)a^2b^2} \\ &= \left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{a-b}{ab}\right) = \left(\frac{a+b}{ab}\right)\left(\frac{a^2-b^2}{ab}\right) = \frac{(a+b)(a^2-b^2)}{a^2b^2} \\ &= \frac{-(a^2-b^2)}{(a+b)a^2b^2} \cdot \frac{a^2b^2}{(a+b)(a^2-b^2)} = \frac{-1}{(a+b)^2} \end{aligned}$$

Jawaban: C

$$\begin{aligned} 2. & \frac{\sqrt{6}}{\sqrt{15}-\sqrt{10}} = \frac{\sqrt{6}}{\sqrt{15}-\sqrt{10}} \times \frac{\sqrt{15}+\sqrt{10}}{\sqrt{15}+\sqrt{10}} \\ &= \frac{\sqrt{6}(\sqrt{15}+\sqrt{10})}{(\sqrt{15})^2-(\sqrt{10})^2} = \frac{\sqrt{90}+\sqrt{60}}{15-10} \\ &= \frac{3\sqrt{10}+2\sqrt{15}}{5} = \frac{3}{5}\sqrt{10} + \frac{2}{5}\sqrt{15} \end{aligned}$$

Jawaban: E

$$\begin{aligned}
 3. \quad {}^2\log \frac{1}{a} = \frac{3}{2} &\Leftrightarrow {}^2\log a^{-1} = \frac{3}{2} \Leftrightarrow {}^2\log a = -\frac{3}{2} \\
 {}^{16}\log b = 5 &\Leftrightarrow {}^{2^4}\log b = 5 \Leftrightarrow \frac{1}{4} {}^2\log b = 5 \Leftrightarrow {}^2\log b = 20 \\
 {}^a\log \frac{1}{b^3} = {}^a\log b^{-3} &= -3 {}^a\log b = -3 \frac{{}^2\log b}{{}^2\log a} = -3 \cdot \frac{20}{-\frac{3}{2}} = 40
 \end{aligned}$$

Jawaban: A

$$4. \quad 2 \log (x-2) \leq \log (2x-1) \Leftrightarrow \log (x-2)^2 \leq \log (2x-1)$$

Syarat pertidaksamaan

$$(x-2)^2 \leq 2x-1 \Leftrightarrow x^2 - 4x + 4 \leq 2x-1$$

$$\Leftrightarrow (x-5)(x-1) \leq 0 \Leftrightarrow 1 \leq x \leq 5 \quad \dots (1)$$

Syarat numerus

$$\bullet \quad x-2 > 0 \Leftrightarrow x > 2 \quad \dots (2)$$

$$\bullet \quad 2x-1 > 0 \Leftrightarrow x > \frac{1}{2} \quad \dots (3)$$

Dari (1), (2), dan (3) diperoleh irisan $2 < x \leq 5$.

Jawaban: B

$$5. \quad f(x) = x^2 + 4 \text{ dan } (g \circ f)(x) = 2x^2 + 9$$

$$(g \circ f)(x) = 2x^2 + 9$$

$$g(f(x)) = 2x^2 + 9$$

$$g(x^2 + 4) = 2(x^2 + 4) + 1$$

$$g(x) = 2x + 1$$

$$g(1) = 2(1) + 1 = 2 + 1 = 3$$

Jawaban: D

$$6. \quad (g \circ f)(x) = g(f(x))$$

$$= g(x+2) = \frac{15}{x+2}$$

Misalkan $(g \circ f)(x) = y$, sehingga

$$y = \frac{15}{x+2} \Leftrightarrow x+2 = \frac{15}{y}$$

$$\Leftrightarrow x = \frac{15}{y} - 2$$

$$\text{Jadi, } (g \circ f)^{-1}(x) = \frac{15}{x} - 2, x \neq 0$$

$$(f^{-1} \circ g^{-1})(x) = (g \circ f)^{-1}(x)$$

$$\text{Sehingga } (f^{-1} \circ g^{-1})(a) = \frac{15}{a} - 2$$

$$\Leftrightarrow 1 = \frac{15}{a} - 2 \Leftrightarrow \frac{15}{a} - 2 = 1 \Leftrightarrow \frac{15}{a} = 3 \Leftrightarrow a = 5$$

Jawaban: C

$$7. \quad \left. \begin{aligned} \alpha + \beta &= \frac{-4}{1} = -4 \\ \alpha \cdot \beta &= \frac{a-4}{1} = a-4 \end{aligned} \right\} \begin{array}{l} \text{Rumus jumlah dan} \\ \text{hasil kali akar-akar} \end{array}$$

$$\alpha + \beta = -4 \Rightarrow \alpha = -4 - \beta$$

$$3\beta = -4 - \beta$$

$$4\beta = -4$$

$$\beta = -1$$

$$\beta = -1 \Rightarrow \alpha = -4 - \beta = -4 - (-1) = -3$$

$$\alpha \cdot \beta = a - 4$$

$$(-3)(-1) = a - 4$$

$$3 = a - 4$$

$$a = 3 + 4 = 7$$

Jawaban: D

$$8. \quad \text{Persamaan mempunyai akar-akar real} \Leftrightarrow D \geq 0$$

$$D \geq 0$$

$$b^2 - 4ac \geq 0$$

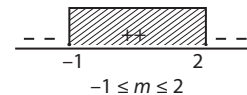
$$4^2 - 4(m-1)(2m) \geq 0$$

$$16 - 8m^2 + 8m \geq 0$$

$$-m^2 + m + 2 \geq 0$$

$$(-m+2)(m+1) \geq 0$$

Pembuat nol fungsi: $m = -1$ dan $m = 2$



Jawaban: A

$$9. \quad (1) \quad \begin{array}{l} p \Rightarrow q \\ q \Rightarrow \sim r \end{array}$$

$$\therefore p \Rightarrow \sim r \equiv r \Rightarrow \sim p \text{ (pernyataan benar)}$$

$$(2) \quad \begin{array}{l} p \Rightarrow q \\ q \Rightarrow s \end{array}$$

$$\therefore p \Rightarrow s$$

$$s \Rightarrow t$$

$$\therefore p \Rightarrow t \equiv \sim p \vee t \text{ (pernyataan benar)}$$

$$(3) \quad \sim p \Rightarrow q$$

$$\sim q$$

$$\therefore p$$

$$p$$

$$p \Rightarrow s$$

$$\therefore s \text{ (pernyataan benar)}$$

$$(4) \quad \sim p \Rightarrow \sim q$$

$$q$$

$$\therefore \text{tidak dapat disimpulkan (pernyataan salah)}$$

Jawaban: A

$$10. \quad f(x) = (k+3)x^2 - 2kx + (k-2), \text{ dengan } a = k+3, b = -2k, c = k-2$$

Syarat definit positif $D < 0$ dan $a > 0$

$$D = b^2 - 4ac < 0$$

$$\bullet \quad D = (-2k)^2 - 4(k+3)(k-2) < 0$$

$$\Leftrightarrow 24 - 4k < 0 \Leftrightarrow k > 6 \quad \dots (1)$$

$$\bullet \quad a = k+3 > 0$$

$$\Leftrightarrow k > -3 \quad \dots (2)$$

Dari (1) dan (2) diperoleh irisan $k > 6$

Jawaban: D

11. $x + y + 2z = k \dots (1)$

$x + 2y + z = k \dots (2)$

$2x + y + z = k \dots (3)$

Persamaan (1) + (2) + (3), diperoleh

$4x + 4y + 4z = 3k \Leftrightarrow x + y + z = \frac{3k}{4} \dots (4)$

Persamaan (1) - (4) $\Rightarrow z = \frac{k}{4}$

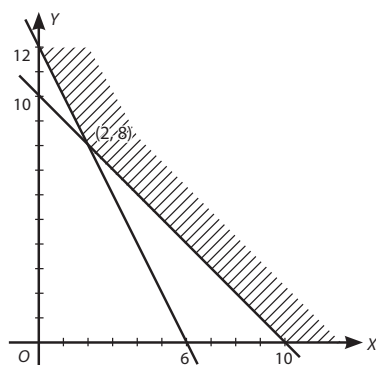
Persamaan (2) - (4) $\Rightarrow y = \frac{k}{4}$

Persamaan (3) - (4) $\Rightarrow x = \frac{k}{4}$

$x^2 + y^2 + z^2 = \left(\frac{k}{4}\right)^2 + \left(\frac{k}{4}\right)^2 + \left(\frac{k}{4}\right)^2 = \frac{3k^2}{16}$

Jawaban: B

12. Gambar daerah dengan kendala $2x + y \geq 12$, $x + y \geq 10$, $x \geq 0$, dan $y \geq 0$.



Titik (2, 8) adalah titik potong garis $2x + y = 12$ dan $x + y = 10$.

Fungsi objektif $f(x, y) = ax + 10y$.

Gunakan metode titik pojok.

$(10, 0) \Rightarrow f = 10a$

$(2, 8) \Rightarrow f = 2a + 80$

$(0, 12) \Rightarrow f = 120$

Agar titik (2, 8) merupakan nilai minimum, maka

$2a + 80 < 10a$ dan $2a + 80 < 120$

$2a + 80 < 10a \Leftrightarrow a > 10 \dots (1)$

$2a + 80 < 120 \Leftrightarrow a < 20 \dots (2)$

Dari (1) dan (2) diperoleh $10 < a < 20$

Jawaban: E

13. $A = \begin{pmatrix} a & 1-b \\ 0 & 1 \end{pmatrix}$

$A^{-1} = \frac{1}{a} \begin{pmatrix} 1 & b-1 \\ 0 & a \end{pmatrix} = \begin{pmatrix} \frac{1}{a} & \frac{b-1}{a} \\ 0 & 1 \end{pmatrix}$

Sehingga

$\begin{pmatrix} \frac{1}{a} & \frac{b-1}{a} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & b \\ 0 & 1 \end{pmatrix}$

Diperoleh kesamaan

$\frac{1}{a} = 2 \Leftrightarrow a = \frac{1}{2}$

$\frac{b-1}{a} = b \Leftrightarrow \frac{b-1}{\frac{1}{2}} = \frac{1}{2} \Leftrightarrow 2b-2 = \frac{1}{2} \Leftrightarrow 2b = \frac{5}{2} \Leftrightarrow b = \frac{5}{4}$

Nilai $2a + b = 2\left(\frac{1}{2}\right) + \frac{5}{4} = 3$

Jawaban: E

14. $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$A^T = A^{-1} \Leftrightarrow \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Sehingga

$a = \frac{d}{ad-bc} \Leftrightarrow ad-bc = \frac{d}{a} \dots (1)$

$d = \frac{a}{ad-bc} \Leftrightarrow ad-bc = \frac{a}{d} \dots (2)$

(1) $\times$ (2), sehingga

$(ad-bc)(ad-bc) = \frac{d}{a} \times \frac{a}{d}$
 $(ad-bc)^2 = 1 \Leftrightarrow ad-bc = \pm 1$

Jawaban: D

15. $U_4 = ar^3 = 48$

$6r^3 = 48$

$r^3 = 8$

$r = 2$

$S_6 = \frac{a(r^6-1)}{r-1} = \frac{6(2^6-1)}{2-1} = \frac{6(64-1)}{1} = 378$

Jawaban: C

16. ${}^y\log(x-1)$, ${}^y\log(x+1)$, ${}^y\log(3x-1)$ merupakan deret aritmetika, sehingga

$2 {}^y\log(x+1) = {}^y\log(x-1) + {}^y\log(3x-1)$

$\Leftrightarrow {}^y\log(x+1)^2 = {}^y\log(x-1)(3x-1)$

$\Leftrightarrow (x+1)^2 = (x-1)(3x-1) \Leftrightarrow 2x^2 - 6x + 0$

$\Leftrightarrow 2x(x-3) = 0 \Leftrightarrow x = 0$ (TM) atau $x = 3$

Selanjutnya

${}^y\log(x-1) + {}^y\log(x+1) + {}^y\log(3x-1) = 6$

$\Leftrightarrow {}^y\log(x-1)(x+1)(3x-1) = {}^y\log y^6$

$\Leftrightarrow (x-1)(x+1)(3x-1) = y^6$

Substitusikan $x = 3$, diperoleh

$(3-1)(3+1)(3 \cdot 3 - 1) = y^6$

$\Leftrightarrow 2 \cdot 4 \cdot 8 = y^6 \Leftrightarrow 2^6 = y^6$

$\Leftrightarrow y = 2$

Jadi, $x + y = 3 + 2 = 5$

Jawaban: D

17. Misalkan y_t = banyaknya bakteri hari ke- t . Sehingga $y_t = Ax^t$, dengan A = suatu konstanta.

$$\text{Hari ke-2} \Rightarrow y_2 = Ax^2 \Leftrightarrow Ax^2 = 36 \quad \dots (1)$$

$$\text{Hari ke-4} \Rightarrow y_4 = Ax^4 \Leftrightarrow Ax^4 = 81 \quad \dots (2)$$

Persamaan (2) : (1), diperoleh

$$\frac{Ax^4}{Ax^2} = \frac{81}{36} \Leftrightarrow x^2 = \frac{9}{4} \Leftrightarrow x = \frac{3}{2}$$

Substitusikan nilai x ini ke (1), diperoleh

$$A\left(\frac{3}{2}\right)^2 = 36 \Leftrightarrow A = 16$$

$$\text{Sehingga } y_t = 16\left(\frac{3}{2}\right)^t.$$

$$\text{Jumlah bakteri awal} \Rightarrow t = 0, \text{ yaitu } y_0 = 16\left(\frac{3}{2}\right)^0 = 16.$$

Jawaban: D

18. $x^2 + x + 1 = 0 \Leftrightarrow x^2 = -x - 1$

$$\begin{array}{c|ccc} & 1 & 0 & a & b \\ -1 & * & -1 & 1 & * \\ -1 & * & * & -1 & 1 \\ \hline & 1 & -1 & a & b+1 \end{array}$$

$$\text{Sisa} = ax + b + 1$$

Karena habis dibagi, maka

$$a = 0$$

$$b + 1 = 0 \Leftrightarrow b = -1$$

$$\text{Nilai } a + b = 0 - 1 = -1$$

Jawaban: B

$$\begin{aligned} 19. \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} &= \lim_{x \rightarrow 4} \frac{(\sqrt{x} - 2)}{(\sqrt{x} - 2)(\sqrt{x} + 2)} \\ &= \lim_{x \rightarrow 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{\sqrt{4} + 2} = \frac{1}{4} \end{aligned}$$

Jawaban: E

$$\begin{aligned} 20. \lim_{x \rightarrow \infty} (\sqrt{9x^2 - 10x + 4} - (3x + 5)) \\ &= \lim_{x \rightarrow \infty} (\sqrt{9x^2 - 10x + 4} - 3x) - 5 \\ &= \lim_{x \rightarrow \infty} (\sqrt{9x^2 - 10x + 4} - \sqrt{9x^2}) - 5 \\ &= \frac{-10 - 0}{2\sqrt{9}} - 5 = \frac{-5}{3} - 5 = -6\frac{2}{3} \end{aligned}$$

Jawaban: E

21. $f(x) = x^2 + 2x - 8$

$$f'(x) = 2x + 2$$

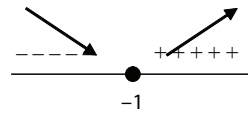
$$f'(x) = 2$$

$$\text{Nilai stasioner } f'(x) = 0$$

$$2x + 2 = 0$$

$$2x = -2$$

$$x = -1$$



Berdasarkan garis bilangan di atas dapat diperoleh fungsi f mencapai nilai minimum untuk nilai $x = -1$.

Sehingga,

$$\begin{aligned} f(-1) &= (-1)^2 + 2(-1) - 8 \\ &= 1 - 2 - 8 = -9 \end{aligned}$$

Jawaban: B

22. Misalkan $u = x - 1$, maka $x = u + 1$ dan $du = dx$

$$\begin{aligned} \int (x+3)(x-1)^5 dx &= \int (u+1+3)u^5 du \\ &= \int (u+4)u^5 du \\ &= \int (u^6 + 4u^5) du \\ &= \frac{1}{7}u^7 + \frac{2}{3}u^6 + C \end{aligned}$$

Kita ganti kembali fungsi u , menjadi

$$\int (x+3)(x-1)^5 dx = \frac{1}{7}(x-1)^7 + \frac{2}{3}(x-1)^6 + C$$

Jawaban: A

23. $y = \frac{4}{3}x\sqrt{x} \Rightarrow \frac{dy}{dx} = 2\sqrt{x}$

$$\begin{aligned} \int_0^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx &= \int_0^2 \sqrt{1 + (2\sqrt{x})^2} dx \\ &= \int_0^2 \sqrt{1 + 4x} dx \end{aligned}$$

$$\text{Misalkan } u = 1 + 4x, \text{ maka } du = 4 dx \text{ atau } dx = \frac{du}{4}$$

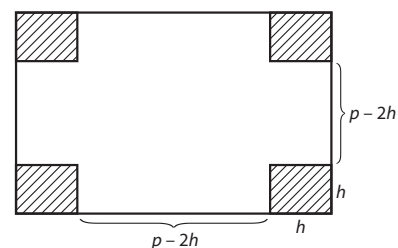
$$\text{Untuk } x = 0 \Rightarrow u = 1 + 4(0) = 1$$

$$\text{Untuk } x = 2 \Rightarrow u = 1 + 4(2) = 9$$

$$\begin{aligned} \int_0^2 \sqrt{1 + 4x} dx &= \int_1^9 \sqrt{u} \frac{du}{4} \\ &= \frac{1}{4} \left[\frac{2}{3} u\sqrt{u} \right]_1^9 = \frac{1}{6} [u\sqrt{u}]_1^9 = \frac{13}{3} \end{aligned}$$

Jawaban: A

24. Perhatikan gambar berikut ini.



Perhatikan bahwa $p - 2h > 0 \Leftrightarrow h < \frac{1}{2}p$

Volume kotak:

$$V = (p - 2h)(p - 2h)(h)$$

$$\Leftrightarrow V = 4ph^3 - 4h^2 + p^2h$$

Volume akan maksimum jika $V' = 0$ (turunan V terhadap h)

$$V' = 12h^2 - 8ph + p^2 = 0$$

$$\Leftrightarrow (6h - p)(2h - p) = 0$$

$$\Leftrightarrow h = \frac{1}{6}p \text{ atau } h = \frac{1}{2}p$$

Karena $h < \frac{1}{2}p$, maka yang memenuhi $h = \frac{1}{6}p$.

Jawaban: C

$$25. \quad \alpha + \beta = \frac{\pi}{6}$$

$$\cos(\alpha + \beta) = \cos \frac{\pi}{6}$$

$$\cos \alpha \cos \beta - \sin \alpha \sin \beta = \frac{1}{2}\sqrt{3}$$

$$\frac{3}{4} - \sin \alpha \sin \beta = \frac{1}{2}\sqrt{3}$$

$$\sin \alpha \sin \beta = \frac{3}{4} - \frac{1}{2}\sqrt{3}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$= \frac{3}{4} + \frac{3}{4} - \frac{1}{2}\sqrt{3} = \frac{6}{4} - \frac{1}{2}\sqrt{3} = \frac{3}{2} - \frac{1}{2}\sqrt{3}$$

Jawaban: D

$$26. \quad \frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 30^\circ} = \frac{b}{\sin 45^\circ}$$

$$\frac{a}{\frac{1}{2}} = \frac{b}{\frac{1}{2}\sqrt{2}}$$

$$a = \frac{b}{\sqrt{2}} = \frac{b}{2}\sqrt{2}$$

$$a + b = 10 \Leftrightarrow \frac{b}{2}\sqrt{2} + b = 10$$

Kedua ruas dikali 2, diperoleh

$$\Leftrightarrow b\sqrt{2} + 2b = 20 \Leftrightarrow b(\sqrt{2} + 2) = 20 \Leftrightarrow b = \frac{20}{\sqrt{2} + 2}$$

Rasionalkan penyebutnya, diperoleh

$$b = \frac{20}{2 + \sqrt{2}} \cdot \frac{2 - \sqrt{2}}{2 - \sqrt{2}} = \frac{20(2 - \sqrt{2})}{4 - 2} = 10(2 - \sqrt{2})$$

Jawaban: C

$$27. \quad 2 \sin^2 x - 7 \sin x + 3 = 0$$

$$\Leftrightarrow (2 \sin x - 1)(\sin x - 3) = 0$$

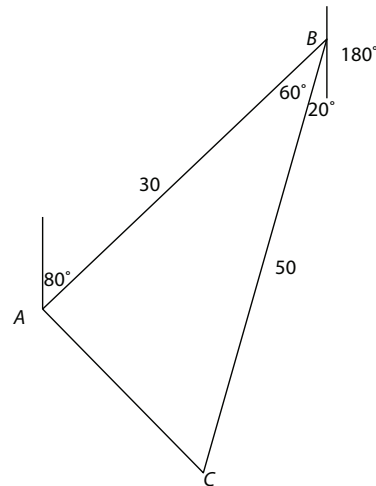
$$\Leftrightarrow \sin x = \frac{1}{2} \text{ atau } \sin x = 3 \text{ (Tidak memenuhi)}$$

$$\sin x = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{6}$$

$$\cos x = \cos \frac{\pi}{6} = \frac{1}{2}\sqrt{3}$$

Jawaban: E

28. Kapal berlayar dari pelabuhan A ke pelabuhan B sejauh 30 km dengan arah 080° . Kemudian dari pelabuhan B ke pelabuhan C sejauh 50 km dengan 200° .



$$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos 60^\circ$$

$$AC^2 = 30^2 + 50^2 - 2 \cdot 30 \cdot 50 \cdot \frac{1}{2}$$

$$AC^2 = 900 + 2.500 - 1.500$$

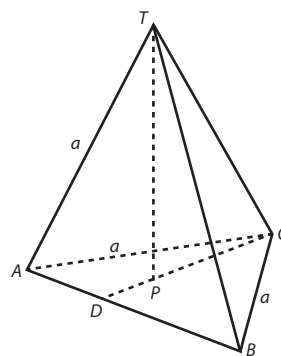
$$AC^2 = 1.900$$

$$AC = \sqrt{1.900} = 10\sqrt{19}$$

Jadi, jarak terdekat antara pelabuhan A dan C adalah $10\sqrt{19}$ km.

Jawaban: C

29.



Tinggi bidang empat $T.ABC$ = jarak T ke bidang ABC .

Perhatikan $\triangle ADC$.

$$AD = \frac{1}{2}a \quad AC = a$$

$$CD = \sqrt{AC^2 - AD^2} = \sqrt{a^2 - \left(\frac{1}{2}a\right)^2} = \frac{1}{2}a\sqrt{3}$$

Karena ABC adalah segitiga samasisi, maka titik P adalah titik beratnya.

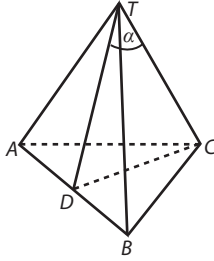
$$PC = \frac{2}{3}CD = \frac{2}{3}\left(\frac{1}{2}a\sqrt{3}\right) = \frac{1}{3}a\sqrt{3}$$

Jarak T ke bidang ABC adalah

$$TP = \sqrt{TC^2 - PC^2} = \sqrt{a^2 - \left(\frac{1}{3}a\sqrt{3}\right)^2} = \frac{1}{3}a\sqrt{6}$$

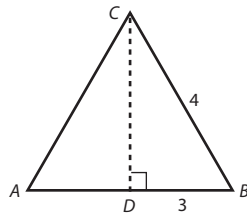
Jawaban: B

30. Perhatikan gambar berikut ini.

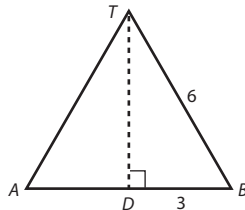


Perhatikan $\triangle ABC$ samakaki.

$$CD = \sqrt{BC^2 - BD^2} = \sqrt{4^2 - 3^2} = \sqrt{7}$$



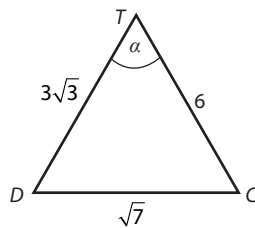
Perhatikan $\triangle ABT$ samasisi.



$$\begin{aligned} TD &= \sqrt{BT^2 - BD^2} \\ &= \sqrt{6^2 - 3^2} \\ &= \sqrt{27} \\ &= 3\sqrt{3} \end{aligned}$$

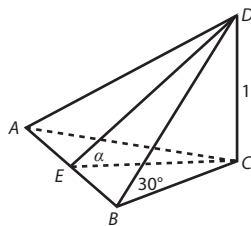
Perhatikan $\triangle TCD$.

$$\begin{aligned} \cos \alpha &= \frac{TC^2 + TD^2 - CD^2}{2 \cdot TC \cdot TD} \\ &= \frac{36 + 27 - 7}{2(6)(3\sqrt{3})} \\ &= \frac{7}{27}\sqrt{3} \end{aligned}$$



Jawaban: E

31.



Perhatikan $\triangle BCD$.

$$\sin 30^\circ = \frac{CD}{BD} \Leftrightarrow BD = \frac{CD}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2$$

$$\tan 30^\circ = \frac{CD}{BC} \Leftrightarrow BC = \frac{CD}{\tan 30^\circ} = \frac{1}{\frac{1}{3}\sqrt{3}} = \sqrt{3}$$

Karena ABC segitiga samasisi, maka $AB = BC = \sqrt{3}$.

$$BE = \frac{1}{2}AB = \frac{1}{2}\sqrt{3}$$

Perhatikan $\triangle BED$ siku-siku di E .

$$DE = \sqrt{BD^2 - BE^2} = \sqrt{2^2 - \left(\frac{1}{2}\sqrt{3}\right)^2} = \frac{1}{2}\sqrt{13}$$

Perhatikan $\triangle CED$ siku-siku di C .

$$EC = \sqrt{DE^2 - CD^2} = \sqrt{\left(\frac{1}{2}\sqrt{13}\right)^2 - 1^2} = \frac{3}{2}$$

$$\tan \alpha = \frac{CD}{EC} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

Jawaban: E

$$32. \quad \overrightarrow{AB} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ p \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

$$\frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AC}|} = \frac{1}{3}\sqrt{6} \Leftrightarrow \frac{2+2-p}{\sqrt{6}} = \frac{1}{3}\sqrt{6}$$

$$\Leftrightarrow 4-p = \frac{1}{3}\sqrt{6} \cdot \sqrt{6}$$

$$\Leftrightarrow 4-p = \frac{1}{3}(6)$$

$$\Leftrightarrow 4-p = 2 \Leftrightarrow p = 2$$

Jawaban: E

33. Misalkan persamaan lingkarannya

$$x^2 + y^2 + 2ax + 2by + c = 0.$$

Persamaan garis singgungnya di titik $(2, -1)$ adalah

$$xx_1 + yy_1 + ax + ax_1 + by + by_1 + c = 0$$

$$\Leftrightarrow 2x - y + ax + 2a + by - b + c = 0$$

$$\Leftrightarrow (a+2)x + (b-1)y + (2a-b+c) = 0$$

Persamaan garis singgung ini harus ekuivalen dengan

$$3x + 2y - 4 = 0, \text{ sehingga}$$

$$a+2=3 \Leftrightarrow a=1$$

$$b-1=2 \Leftrightarrow b=3$$

$$2a-b+c=-4 \Leftrightarrow 2(1)-3+c=-4 \Leftrightarrow c=-3$$

Jadi, persamaan lingkarannya adalah

$$x^2 + y^2 + 2x + 6y - 3 = 0$$

Titik pusatnya $(-1, -3)$.

Jawaban: B

34. Persamaan lingkaran dengan pusat $(-1, 3)$ dan jari-jari 1 adalah

$$(x + 1)^2 + (y - 3)^2 = 1 \quad \dots (1)$$

$$\text{Persamaan garis } ax + y = 0 \Leftrightarrow y = -ax \quad \dots (2)$$

Substitusikan (2) ke (1) diperoleh

$$(x + 1)^2 + (-ax - 3)^2 = 1$$

$$\Leftrightarrow (a^2 + 1)x^2 + (6a + 2)x + 9 = 0$$

$$\text{Syarat menyinggung} \Rightarrow D = 0$$

$$D = (6a + 2)^2 - 4(a^2 + 1)(9) = 0$$

$$\Leftrightarrow 24a - 32 = 0 \Leftrightarrow a = \frac{4}{3}$$

Jawaban: B

35. Diketahui $T_1 = \begin{pmatrix} 5 & 3 \\ -1 & 2 \end{pmatrix}$ dan $T_2 = \begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix}$.

$$\begin{pmatrix} -9 \\ 7 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} -9 \\ 7 \end{pmatrix} = \begin{pmatrix} -1 & -3 \\ -5 & 11 \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix} \Leftrightarrow \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} -1 & -3 \\ -5 & 11 \end{pmatrix}^{-1} \begin{pmatrix} -9 \\ 7 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} m \\ n \end{pmatrix} = \frac{1}{-26} \begin{pmatrix} 11 & 3 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} -9 \\ 7 \end{pmatrix} \Leftrightarrow \begin{pmatrix} m \\ n \end{pmatrix} = \frac{1}{-26} \begin{pmatrix} -78 \\ -52 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\text{Nilai } m + n = 3 + 2 = 5$$

Jawaban: E

36. Letak median pada data ke- $\frac{1}{2}n = \frac{1}{2}(30) = 15$, yaitu di kelas ke-2

$$T_b = 49,5 \quad \Sigma f = 7 \quad f_{Me} = 9 \quad c = 10$$

$$Me = T_b + \frac{\frac{1}{2}n - \Sigma f}{f_{Me}} c$$

$$\Leftrightarrow Me = 49,5 + \frac{15 - 7}{9}(10) = 49,5 + \frac{80}{9}$$

Jawaban: A

$$37. n = \sum f_i = 40$$

Kuartil bawah = Q_1

Kelas Q_1 berada pada data ke-

$$\frac{1}{4} \times n = \frac{1}{4} \times 40 = 10 \text{ (kelas ke-2)}$$

$$T_b = 41 - 0,5 = 40,5$$

Panjang kelas, $I = 10$

$$Q_1 = T_b + \left(\frac{\frac{1}{4}n - F_k}{f_{Q_1}} \right) I = 40,5 + \left(\frac{10 - 5}{9} \right) 10 = 40,5 + \left(\frac{5}{9} \right) 10 = 40,5 + \frac{50}{9} = 40,5 + 5,56 = 46,06$$

Jawaban: B

38. Kemungkinan susunan

$$3P \ 3W \Rightarrow C_3^5 \cdot C_3^4 = 10 \cdot 4 = 40 \text{ cara}$$

$$4P \ 2W \Rightarrow C_4^5 \cdot C_2^4 = 5 \cdot 6 = 30 \text{ cara}$$

$$5P \ 1W \Rightarrow C_5^5 \cdot C_1^4 = 1 \cdot 4 = 4 \text{ cara}$$

$$\text{Banyaknya cara} = 40 + 30 + 4 = 74$$

Jawaban: D

39. Nomor 1 sampai 5 harus dikerjakan, berarti 5 soal harus dikerjakan.

Siswa memilih 3 soal dari 5 soal yang tersisa, artinya kombinasi 3 dari 5.

$$C_{(5,3)} = \frac{5!}{3!(5-3)!} = \frac{5 \cdot 4 \cdot 3!}{3!(2 \cdot 1)} = 10$$

Jadi, banyak pilihan soal yang dapat dikerjakan siswa adalah 10 cara.

Jawaban: D

$$40. n(\text{Voli}) = 11 \quad P(\text{Voli}) = \frac{11}{40}$$

$$n(\text{Basket}) = 15 \quad P(\text{Basket}) = \frac{15}{40}$$

$$n(\text{Voli} \cap \text{Basket}) = 5 \quad P(\text{Voli} \cap \text{Basket}) = \frac{5}{40}$$

$$P(\text{Voli} \cap \text{Basket}) = P(\text{Voli}) + P(\text{Basket}) - P(\text{Voli} \cap \text{Basket}) = \frac{11}{40} + \frac{15}{40} - \frac{5}{40} = \frac{21}{40}$$

Jawaban: E

PEMBAHASAN DAN KUNCI JAWABAN PEMANTAPAN PAKET 3

MATEMATIKA IPA

$$1. \frac{(2a)^3(2a)^{-\frac{2}{3}}}{(16a^4)^{\frac{1}{3}}} = \frac{(2a)^{\frac{7}{3}}}{(2^4 a^4)^{\frac{1}{3}}} = \frac{2^{\frac{7}{3}} a^{\frac{7}{3}}}{2^{\frac{4}{3}} a^{\frac{4}{3}}} = 2a$$

Jawaban: E

$$2. \frac{p}{q} = \frac{(x^{\frac{3}{2}} + x^{\frac{1}{2}})(x^{\frac{1}{2}} - x^{-\frac{1}{2}})}{(x^{\frac{1}{2}} + x^{-\frac{1}{2}})(x - x^{\frac{1}{2}})} \\ = \frac{x^{\frac{3}{2}}(x+1)x^{-\frac{1}{2}}(x^{\frac{2}{2}}-1)}{x^{-\frac{1}{2}}(x+1)x^{\frac{1}{2}}(x^{\frac{2}{2}}-1)} = \frac{x^{\frac{3}{2}}x^{-\frac{1}{2}}}{x^{-\frac{1}{2}}x^{\frac{1}{2}}} = \frac{x^{\frac{3}{2}-\frac{1}{2}}}{x^{\frac{1}{2}-\frac{1}{2}}} = 1$$

Jawaban: C

$$3. \frac{\frac{1}{5}\log 3 \cdot \frac{3}{5}\log 125 \cdot \frac{5}{5}\log \frac{1}{125}}{(\frac{2}{5}\log 4)^2 - \frac{1}{5}\log 9} = \frac{\frac{1}{5}\log 3 \cdot \frac{3}{5}\log 5^3 \cdot \frac{5}{5}\log 5^{-3}}{(\frac{2}{5}\log 2^2)^2 - \frac{1}{5}\log 3^2} \\ = \frac{\frac{1}{5} \cdot 3 \cdot (-3)}{2^2 - \frac{1}{5} \cdot 2} = \frac{9}{4+2} = \frac{9}{6} = \frac{3}{2}$$

Jawaban: E

$$4. 3^{2x} - 6 \cdot 3^x < 27 \Leftrightarrow (3^x)^2 - 6 \cdot 3^x - 27 < 0$$

Misalkan $m = 3^x$, sehingga

$$m^2 - 6m - 27 < 0$$

Menentukan batasnya

$$m^2 - 6m - 27 = 0 \Leftrightarrow (m-9)(m+3) = 0$$

$$\Leftrightarrow m = 9 \text{ atau } m = -3$$

$$\text{Untuk } m = 9 \Rightarrow 3^x = 9 \Leftrightarrow x = 2$$

Untuk $m = -3 \Rightarrow 3^x = -3 \Leftrightarrow$ tidak ada nilai x yang memenuhi

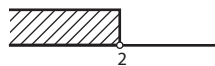
Pilih titik uji $x = 0$, masukkan ke

pertidaksamaan, diperoleh

$$3^0 - 6 \cdot 3^0 < 27$$

$$\Leftrightarrow 1 - 6 < 27 \text{ (benar)}$$

Sehingga pertidaksamaan dipenuhi oleh $x < 2$.



Jawaban: C

$$5. (f \circ g)(x) = \frac{1}{\sqrt{x^2 + 6x + 7}} \Leftrightarrow f(g(x)) = \frac{1}{\sqrt{x^2 + 6x + 7}}$$

$$\Leftrightarrow \frac{1}{\sqrt{\{g(x)\}^2 - 2}} = \frac{1}{\sqrt{x^2 + 6x + 7}}$$

$$\Leftrightarrow \sqrt{\{g(x)\}^2 - 2} = \sqrt{x^2 + 6x + 7}$$

Kuadratkan kedua ruas diperoleh

$$\{g(x)\}^2 - 2 = x^2 + 6x + 7 \Leftrightarrow \{g(x)\}^2 = x^2 + 6x + 9$$

$$\Leftrightarrow \{g(x)\}^2 = (x+3)^2 \Leftrightarrow g(x) = x+3$$

$$\Leftrightarrow g(x+2) = (x+2) + 3 = x+5$$

Jawaban: E

$$6. f^{-1}(x) = \frac{x-1}{5} \text{ dan } g^{-1}(x) = \frac{3x-1}{2}$$

$$(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$$

$$= g^{-1}(f^{-1}(x)) = g^{-1}\left(\frac{x-1}{5}\right)$$

$$= \frac{3\left(\frac{x-1}{5}\right) - 1}{2} = \frac{3x-3-5}{2}$$

$$= \frac{3x-8}{2}$$

$$(f \circ g)^{-1}(6) = \frac{3(6)-8}{2} = \frac{18-8}{2} = \frac{10}{2} = 5$$

Jawaban: C

7. Akar-akar persamaan $x^2 + ax - 4 = 0$ adalah x_1 dan x_2 , sehingga

$$x_1 + x_2 = -a \text{ dan } x_1 x_2 = -4$$

$$x_1^2 - 2x_1 x_2 + x_2^2 = 8a \Leftrightarrow x_1^2 + x_2^2 - 2x_1 x_2 = 8a$$

$$\Leftrightarrow (x_1 + x_2)^2 - 2x_1 x_2 - 2x_1 x_2 = 8a$$

$$\Leftrightarrow (x_1 + x_2)^2 - 4x_1 x_2 = 8a \Leftrightarrow (-a)^2 - 4(-4) - 8a = 0$$

$$\Leftrightarrow a^2 - 8a + 16 = 0 \Leftrightarrow (a-4)^2 = 0 \Leftrightarrow a = 4$$

Jawaban: B

8. x_1 dan x_2 adalah akar-akar persamaan kuadrat

$$x^2 - 2x - 1 = 0, \text{ sehingga,}$$

$$x_1 + x_2 = 2 \text{ dan } x_1 x_2 = -1$$

Selanjutnya

$$x_1^2 + x_2^2 + x_1 + x_2^2 = x_1 + x_2 + (x_1 + x_2)^2 - 2x_1 x_2$$

$$= 2 + 2^2 - 2(-1) = 8$$

$$(x_1^2 + x_2^2)(x_1 + x_2^2) = x_1^3 + x_1^2 x_2^2 + x_1 x_2 + x_2^3$$

$$= (x_1 + x_2)^3 - 3x_1 x_2 (x_1 + x_2) + (x_1 x_2)^2 + x_1 x_2$$

$$= 2^3 - 3(-1)(2) + (-1)^2 + (-1) = 14$$

Persamaan kuadrat yang baru adalah $x^2 - 8x + 14 = 0$.

Jawaban: A

9. Premis 1: Jika kamu sering berlatih soal, maka kamu bisa mengerjakan soal.

Premis 2: Jika kamu tidak lulus ujian, maka kamu tidak bisa mengerjakan soal.

Kedua premis di atas dapat dimisalkan sebagai berikut.

$$P_1: p \Rightarrow q$$

$$P_2: \sim r \Rightarrow \sim q \equiv q \Rightarrow r$$

$\therefore p \Rightarrow r$ negasinya adalah

$$\sim(p \Rightarrow r) \equiv p \wedge \sim r$$

$(p \wedge \sim r)$: Kamu sering berlatih soal dan kamu tidak lulus ujian.

Jawaban: D

10. $y = px^2 + (p-3)x + 2$, dengan $a = p$, $b = (p-3)$, dan $c = 2$

Ordinat titik balik = $\frac{D}{-4a}$, dengan $D = b^2 - 4ac$

$$\frac{D}{-4a} = \frac{b^2 - 4ac}{-4a}$$

$$p = \frac{(p-3)^2 - 4(p)(2)}{-4(p)}$$

$$p = \frac{p^2 - 6p + 9 - 8p}{-4p}$$

$$-4p^2 = p^2 - 14p + 9$$

$$5p^2 - 14p + 9 = 0$$

$$(5p-9)(p-1) = 0$$

Diperoleh: $p = \frac{9}{5}$ atau $p = 1$

Jawaban: E

11. Persamaan (1) : $2x - z = 2$

Persamaan (2) : $x + 2y = 4$

Persamaan (3) dikali 2 : $\frac{2y + 2z = 2}{3x + 4y + z = 8} +$

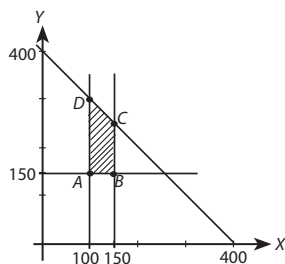
Jawaban: E

12. Misalkan: sepatu laki-laki = x dan sepatu wanita = y
Model matematikanya:

$$100 \leq x \leq 150, y \geq 150, x + y \leq 400$$

Fungsi tujuan: Maksimumkan $f(x, y) = 1.000x + 500y$

Sketsa gambar:



Titik Pojok	$f(x, y) = 1.000x + 500y$
A(100, 150)	$100.000 + 75.000 = 175.000$
B(150, 150)	$150.000 + 75.000 = 225.000$
C(150, 250)	$150.000 + 125.000 = 275.000$
D(100, 300)	$100.000 + 150.000 = 250.000$

← maksimal

Jadi, keuntungan terbesar adalah Rp275.000,00.

Jawaban: C

$$13. \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} 2a & 3b \\ -2 & c \end{pmatrix} + \begin{pmatrix} b & 2c \\ 4 & -4 \end{pmatrix}$$

$$\begin{pmatrix} -1+4 & 3-10 \\ -4+6 & 12-15 \end{pmatrix} = \begin{pmatrix} 2a+b & 3b+2c \\ 2 & c-4 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -7 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 2a+b & 3b+2c \\ 2 & c-4 \end{pmatrix}$$

Diperoleh:

$$-3 = c - 4 \Leftrightarrow c = 1$$

$$3b + 2c = -7$$

$$3b + 2(1) = -7$$

$$3b = -9 \Leftrightarrow b = -3$$

$$2a + b = 3$$

$$2a + (-3) = 3$$

$$2a - 3 = 3$$

$$2a = 6 \Leftrightarrow a = 3$$

Jawaban: D

$$14. \begin{pmatrix} -1 & d \\ -b & 3 \end{pmatrix} + \begin{pmatrix} 4 & -5 \\ -3 & b \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 2c & 1 \\ c & a+1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & d-5 \\ -b-3 & 3+b \end{pmatrix} = \begin{pmatrix} 3c & 1-a \\ -5c & 3a-1 \end{pmatrix}$$

Diperoleh persamaan

$$3 = 3c \quad \dots (1)$$

$$-b - 3 = -5c \quad \dots (2)$$

$$3 + b = 3a - 1 \quad \dots (3)$$

$$d - 5 = 1 - a \quad \dots (4)$$

Dari (1) diperoleh $c = 1$.

Substitusikan $c = 1$ ke (2), diperoleh

$$-b - 3 = -5(1) \Leftrightarrow b = 2$$

Substitusikan $b = 2$ ke (3), diperoleh

$$3 + 2 = 3a - 1 \Leftrightarrow a = 2$$

Substitusikan $a = 2$ ke (4), diperoleh

$$d - 5 = 1 - 2 \Leftrightarrow d = 4$$

$$\text{Matriks } \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 1 & 4 \end{pmatrix}$$

$$\text{Determinan } \begin{pmatrix} 2 & 2 \\ 1 & 4 \end{pmatrix} = (2)(4) - (2)(1) = 6$$

Jawaban: E

15. Misalkan barisannya adalah

$$a, a + b, a + 2b, a + 3b, a + 4b.$$

$$S_5 = 5a + 10b = 20 \Leftrightarrow a + 2b = 4 \quad \dots (*)$$

Selanjutnya,

$$(U_1 - U_3)(U_2 - U_3)(U_4 - U_3)(U_5 - U_3) = 324$$

$$\{a - (a + 2b)\}\{(a + b) - (a + 2b)\}\{(a + 3b) - (a + 2b)\}$$

$$\{(a + 4b) - (a + 2b)\} = 324$$

$$(-2b)(-b)(b)(2b) = 324 \Leftrightarrow 4b^4 = 324 \Leftrightarrow b^4 = 81$$

$$\Leftrightarrow b = 3 \text{ atau } b = -3$$

- Untuk $b = 3 \Rightarrow$ persamaan (*) menjadi
 $a + 2(3) = 4 \Leftrightarrow a = -2$

$$\text{Sehingga } S_8 = \frac{8}{2} (2a + 7b) = 4(2 \cdot (-2) + 7 \cdot 3) = 68$$

- Untuk $b = -3 \Rightarrow$ persamaan (*) menjadi
 $a + 2(-3) = 4 \Leftrightarrow a = 10$

$$\text{Sehingga } S_8 = \frac{8}{2} (2a + 7b) = 4(2 \cdot 10 + 7(-3)) = -4$$

Jadi, nilai $S_8 = -4$ atau 68.

Jawaban: A

16. $U_1 = a = 35.000$

$$b = 7.000$$

$$S_n = 882.000$$

$$S_n = \frac{n}{2} (2a + (n-1)b)$$

$$882.000 = \frac{n}{2} (2 \cdot 35.000 + (n-1)7.000)$$

$$1.764.000 = n(70.000 + 7.000n - 7.000)$$

$$1.764.000 = 63.000n + 7.000n^2$$

$$252 = 9n + n^2$$

$$n^2 + 9n - 252 = 0$$

$$(n+21)(n-12) = 0$$

Diperoleh $n = -21$ atau $n = 12$

Jadi, lama Deni harus menabung adalah 12 bulan.

Jawaban: B

17. Misalkan massa awal 80 gram

Setelah 15 menit massanya menjadi 40 gram, dan seterusnya

Masalah tersebut dapat diselesaikan menggunakan konsep deret geometri.

$$U_1 = a = 40 \text{ gram}$$

$$r = \frac{1}{2}$$

$$n = \frac{1 \text{ jam } 30 \text{ menit}}{15 \text{ menit}} = \frac{90 \text{ menit}}{15 \text{ menit}} = 6$$

Sehingga,

$$U_n = a \cdot r^{n-1}$$

$$U_6 = 40 \cdot \left(\frac{1}{2}\right)^{6-1}$$

$$U_6 = 40 \cdot \left(\frac{1}{2}\right)^5$$

$$U_6 = 40 \cdot \frac{1}{32} = 1,25$$

$$\text{Massa yang meluruh} = 80 - 1,25 = 78,75$$

Jadi, massa unsur yang meluruh selama 1 jam 30 menit adalah 78,75 gram.

Jawaban: D

18. $p(x)$ dibagi $(x+3)$ bersisa 2, berarti $p(-3) = 2$

$$p(-3) = (-3)^4 + 5(-3)^3 + 9(-3)^2 + 13(-3) + a = 2$$

$$81 - 135 + 81 - 39 + a = 2$$

$$-12 + a = 2$$

$$a = 14$$

$$\begin{array}{r|rrrrr} -1 & 1 & 5 & 9 & 13 & 14 \\ & \downarrow & & & & \\ & 1 & 4 & 5 & 8 & \boxed{6} \end{array} \rightarrow \text{sisa}$$

Jawaban: E

19. $\lim_{x \rightarrow 2} \frac{x\sqrt{x} - 2\sqrt{x} - 2\sqrt{2} + x\sqrt{2}}{\sqrt{x} - \sqrt{2}}$

$$= \lim_{x \rightarrow 2} \frac{(x-2)\sqrt{x} + (x-2)\sqrt{2}}{\sqrt{x} - \sqrt{2}}$$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(\sqrt{x} + \sqrt{2})}{\sqrt{x} - \sqrt{2}}$$

$$= \lim_{x \rightarrow 2} \frac{(\sqrt{x} + \sqrt{2})(\sqrt{x} - \sqrt{2})(\sqrt{x} + \sqrt{2})}{\sqrt{x} - \sqrt{2}}$$

$$= \lim_{x \rightarrow 2} (\sqrt{x} + \sqrt{2})^2 = (\sqrt{2} + \sqrt{2})^2 = (2\sqrt{2})^2 = 8$$

Jawaban: D

20. $\lim_{x \rightarrow \infty} (\sqrt{4x^2 + 3x + 1} - 2x - 1)$

$$= \lim_{x \rightarrow \infty} (\sqrt{4x^2 + 3x + 1} - 2x) - 1$$

$$= \lim_{x \rightarrow \infty} (\sqrt{4x^2 + 3x + 1} - \sqrt{4x^2}) - 1$$

$$= \frac{3-0}{2\sqrt{4}} - 1 = \frac{3}{4} - 1 = -\frac{1}{4}$$

Jawaban: D

21. $y = \cos^3(3-2x)$

Misalkan $u = 3 - 2x$, maka $\frac{du}{dx} = -2$

$v = \cos u$, maka $\frac{dv}{du} = -\sin u$

$y = v^3$, maka $\frac{dy}{dv} = 3v^2$

Sehingga,

$$y' = \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= 3v^2 \cdot (-\sin u) \cdot (-2)$$

$$= 6\cos^2 u \cdot \sin u$$

$$= 6\cos^2(3-2x) \sin(3-2x)$$

$$= 3\cos(3-2x) \sin(6-4x)$$

Jawaban: E

22. $\int 3x \cos 2x \, dx$

Misalkan $u = 3x$, maka $du = 3 \, dx$

$dv = \cos 2x \, dx$, maka $v = \frac{\sin 2x}{2}$

Sehingga,

$$\begin{aligned}\int 3x \cos 2x \, dx &= \frac{3x \sin 2x}{2} - \int \frac{3 \sin 2x}{2} \, dx \\ &= \frac{3}{2} x \sin 2x - \frac{3}{2} \left(-\frac{\cos 2x}{2} \right) + C \\ &= \frac{3}{2} x \sin 2x + \frac{3}{4} \cos 2x + C\end{aligned}$$

Jawaban: D

23. Misalkan $u = x + 2$, maka $x = u - 2$ dan $du = dx$

Untuk $x = -1 \Rightarrow u = 1$

Untuk $x = a \Rightarrow u = a + 2$

$$\begin{aligned}\int_{-1}^a \frac{x+1}{(x+2)^4} \, dx &= \frac{10}{81} \Leftrightarrow \int_1^{a+2} \frac{u-2+1}{u^4} \, du = \frac{10}{81} \\ \Leftrightarrow \int_1^{a+2} \frac{u-1}{u^4} \, du &= \frac{10}{81} \Leftrightarrow \int_1^{a+2} \left(\frac{1}{u^3} - \frac{1}{u^4} \right) \, du = \frac{10}{81} \\ \Leftrightarrow \int_1^{a+2} (u^{-3} - u^{-4}) \, du &= \frac{10}{81} \Leftrightarrow \left[-\frac{1}{2} u^{-2} + \frac{1}{3} u^{-3} \right]_1^{a+2} = \frac{10}{81} \\ \Leftrightarrow \left(-\frac{1}{2(a+2)^2} + \frac{1}{3(a+2)^3} \right) &- \left(-\frac{1}{2} + \frac{1}{3} \right) = \frac{10}{81} \\ \Leftrightarrow \frac{-3(a+2)+2}{6(a+2)^3} &= \frac{10}{81} - \frac{1}{6} \Leftrightarrow \frac{-3a-4}{6(a+2)^3} = -\frac{7}{162} \\ \Leftrightarrow 42(a+2)^3 &= -162(-3a-4) \\ \Leftrightarrow 42a^3 + 252a^2 + 18a - 312 &= 0 \\ \Leftrightarrow 6(a-1)(7a^2 + 49a + 52) &= 0 \Leftrightarrow a = 1\end{aligned}$$

Jawaban: D

24. $p + q = 6 \Rightarrow q = 6 - p$

Substitusikan $q = 6 - p$ ke $2p^2 + q^2$ diperoleh

$$\begin{aligned}2p^2 + q^2 &= 2p^2 + (6 - p)^2 \\ &= 2p^2 + 36 - 12p + p^2 = 3p^2 - 12p + 36\end{aligned}$$

Turunannya = 0 $\Leftrightarrow 6p - 12 = 0$

$$\Leftrightarrow p = 2$$

Nilai $q = 6 - 2 = 4$

Sehingga $2p^2 + q^2 = 2(2)^2 + 4^2 = 24$

Jawaban: D

25. $\alpha + \beta = \frac{2}{3}\pi = 120^\circ$

Karena $\sin \alpha = 2 \sin \beta$, sehingga dengan mudah kita dapat bahwa $\alpha = 90^\circ$ dan $\beta = 30^\circ$.

$$\tan(\alpha - \beta) = \tan(90^\circ - 30^\circ) = \tan 60^\circ = \sqrt{3}$$

Jawaban: D

26. $\cos 40^\circ + \cos 80^\circ + \cos 160^\circ$

$$\begin{aligned}&= \left(2 \cos \frac{1}{2}(120^\circ) \cos \frac{1}{2}(-40^\circ) \right) + \cos 160^\circ \\ &= 2 \cos 60^\circ \cos(-20^\circ) + \cos 160^\circ = 2 \cdot \frac{1}{2} \cos 20^\circ + \cos 160^\circ \\ &= \cos 20^\circ + \cos 160^\circ = 2 \cos \frac{1}{2}(180^\circ) \cos \frac{1}{2}(-140^\circ) \\ &= 2 \cos 90^\circ \cos 70^\circ = 2 \cdot 0 \cdot \cos 70^\circ = 0\end{aligned}$$

Jawaban: C

27. $\cos x + \sin x = \frac{1}{2}\sqrt{6}$

$$\Rightarrow a = 1 \text{ dan } b = 1$$

$$p = \sqrt{a^2 + b^2} = \sqrt{2}$$

$$\tan \alpha = \frac{b}{a} = 1$$

$$\Leftrightarrow \alpha = \frac{\pi}{4} \text{ atau } \frac{5\pi}{4}$$

dipilih $\alpha = \frac{\pi}{4}$ karena

titik (1, 1) berada di kuadran I.

Sehingga,

$$\cos x + \sin x = \frac{1}{2}\sqrt{6}$$

$$\Leftrightarrow \sqrt{2} \cos \left(x - \frac{\pi}{4} \right) = \frac{1}{2}\sqrt{6}$$

$$\Leftrightarrow \cos \left(x - \frac{\pi}{4} \right) = \frac{1}{2}\sqrt{3} = \cos \frac{\pi}{6}$$

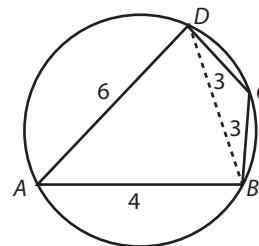
$$x - \frac{\pi}{4} = \frac{\pi}{6} + k \cdot 2\pi \text{ atau } x - \frac{\pi}{4} = -\frac{\pi}{6} + k \cdot 2\pi$$

$$\Leftrightarrow x = \frac{5}{12}\pi + k \cdot 2\pi \text{ atau } x = \frac{1}{12}\pi + k \cdot 2\pi$$

$$k = 0 \Rightarrow x = \frac{5}{12}\pi \text{ atau } x = \frac{1}{12}\pi$$

Jawaban: C

- 28.



Diketahui: $AD = 6$, $AB = 4$, $CD = BC = 3$

Perhatikan $\triangle ABD$.

$$\begin{aligned}BD^2 &= AB^2 + AD^2 - 2 AB \cdot AD \cos \angle BAD \\ &= 16 + 36 - 48 \cos \angle BAD \\ &= 52 - 48 \cos \angle BAD \quad \dots(1)\end{aligned}$$

Perhatikan $\triangle BCD$.

$$\begin{aligned}BD^2 &= BC^2 + CD^2 - 2 BC \cdot CD \cos \angle BCD \\ &= 9 + 9 - 18 \cos \angle BCD \\ &= 18 - 18 \cos \angle BCD \quad \dots(2)\end{aligned}$$

(1) = (2), diperoleh

$$52 - 48 \cos \angle BAD = 18 - 18 \cos \angle BCD$$

$$48 \cos \angle BAD - 18 \cos \angle BCD = 34$$

$$\Leftrightarrow 24 \cos \angle BAD - 9 \cos \angle BCD = 17 \quad \dots(3)$$

Karena $\angle BAD + \angle BCD = 180^\circ$ (sifat segiempat tali busur lingkaran)

$$\Leftrightarrow \angle BCD = 180^\circ - \angle BAD$$

$$\Leftrightarrow \cos \angle BCD = \cos(180^\circ - \angle BAD) = -\cos \angle BAD$$

Sehingga persamaan (3) menjadi

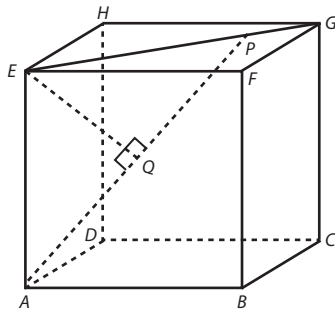
$$24 \cos \angle BAD - 9 (-\cos \angle BAD) = 17$$

$$\Leftrightarrow 33 \cos \angle BAD = 17$$

$$\Leftrightarrow \cos \angle BAD = \frac{17}{33}$$

Jawaban: E

29. Perhatikan gambar berikut ini.



Misalkan rusuk kubus = x . Sehingga $EG = x\sqrt{2}$.

$$EP = \frac{3}{4}EG = \frac{3}{4}x\sqrt{2}$$

Jarak titik E ke AP adalah panjang $EQ = a$.

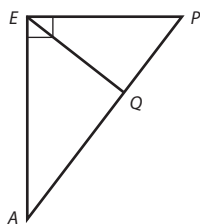
Perhatikan $\triangle APE$ yang siku-siku di E .

$$AP = \sqrt{\left(\frac{3}{4}x\sqrt{2}\right)^2 + x^2} = \frac{1}{4}x\sqrt{34}$$

$$AP \cdot EQ = EA \cdot EP$$

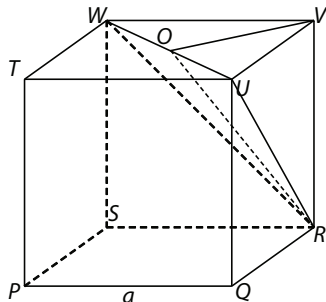
$$\frac{1}{4}x\sqrt{34} \cdot a = x \cdot \frac{3}{4}x\sqrt{2}$$

$$x = \frac{\frac{1}{4}a\sqrt{34}}{\frac{3}{4}\sqrt{2}} = \frac{a}{3}\sqrt{17}$$



Jawaban: C

30. Perhatikan kubus $PQRS.TUVW$ di bawah ini.

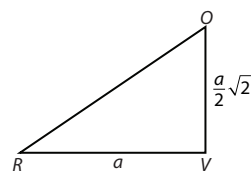


$$UR = a\sqrt{2}$$

$$OV = \frac{1}{2}UR = \frac{a}{2}\sqrt{2}$$

$$\angle(VR, RUW) = \angle(VR, RO) = \beta$$

Perhatikan segitiga OVR siku-siku di V .



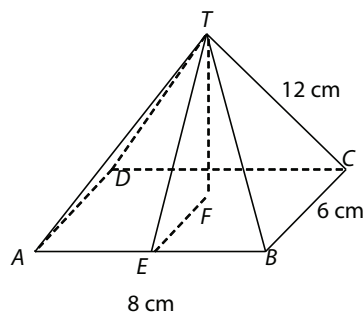
$$RO = \sqrt{RV^2 + OV^2} = \sqrt{a^2 + \left(\frac{a}{2}\sqrt{2}\right)^2}$$

$$= \sqrt{a^2 + \frac{a^2}{2}} = \sqrt{\frac{3}{2}a^2} = \frac{a}{2}\sqrt{6}$$

$$\cos \beta = \frac{RV}{RO} = \frac{a}{\frac{a}{2}\sqrt{6}} = \frac{2}{\sqrt{6}} = \frac{1}{3}\sqrt{6}$$

Jawaban: A

31. Perhatikan limas $T.ABCD$ di bawah ini.

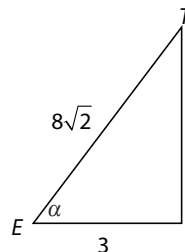


$$\angle(TAB, ABCD) = \angle(TE, EF) = \alpha$$

$$TE = \sqrt{TB^2 - EB^2} = \sqrt{12^2 - 4^2}$$

$$= \sqrt{144 - 16} = \sqrt{128} = 8\sqrt{2}$$

Perhatikan segitiga TEF yang siku-siku di F .



$$\cos \alpha = \frac{EF}{ET} = \frac{3}{8\sqrt{2}} = \frac{3}{16}\sqrt{2}$$

Jawaban: B

$$32. \overrightarrow{AC} = C - A = (1, -3, -3)$$

$$\overrightarrow{AB} = B - A = (-1, -1, -2)$$

$$\Rightarrow |\overrightarrow{AB}| = \sqrt{(-1)^2 + (-1)^2 + (-2)^2} = \sqrt{6}$$

$$\text{Panjang proyeksi } \overrightarrow{AC} \text{ pada } \overrightarrow{AB} = \frac{\overrightarrow{AC} \cdot \overrightarrow{AB}}{|\overrightarrow{AB}|}$$

$$\frac{\overrightarrow{AC} \cdot \overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{-1 + 3 + 6}{\sqrt{6}} = \frac{8}{\sqrt{6}} = \frac{8}{6}\sqrt{6} = \frac{4}{3}\sqrt{6}$$

Jawaban: B

33. Pusat $(3, -3)$ dan menyinggung garis $y = -5$, maka $r = 2$

Persamaan lingkaran:

$$\begin{aligned}(x-3)^2 + (y+3)^2 &= 2^2 \\ x^2 - 6x + 9 + y^2 + 6y + 9 &= 4 \\ x^2 + y^2 - 6x + 6y + 14 &= 0\end{aligned}$$

Jawaban: D

34. $x^2 + y^2 + 4x - 2y - 4 = 0$

$$\begin{aligned}x^2 + 4x + 4 + y^2 - 2y + 1 &= 4 + 1 + 4 \\ (x+2)^2 + (y-1)^2 &= 9\end{aligned}$$

Diperoleh pusat $(-2, 1)$ dan $r = 3$

Gradien dari garis $2x - y + 7 = 0$ adalah $m = 2$, karena tegak lurus maka

$$m_1 = -\frac{1}{m} = -\frac{1}{2}$$

Persamaan garis singgung:

$$\begin{aligned}y - y_p &= m(x - x_p) \pm r\sqrt{1+m^2} \\ y - 1 &= \left(-\frac{1}{2}\right)(x+2) \pm 3\sqrt{1+\left(-\frac{1}{2}\right)^2} \\ y - 1 &= \frac{x+2}{-2} \pm 3\sqrt{1+\frac{1}{4}} \\ y - 1 &= \frac{x+2}{-2} \pm \frac{3\sqrt{5}}{2} \\ 2y - 2 &= -x - 2 \pm 3\sqrt{5}\end{aligned}$$

$$2y = -x \pm 3\sqrt{5} \Rightarrow \begin{cases} x + 2y - 3\sqrt{5} = 0 \\ x + 2y + 3\sqrt{5} = 0 \end{cases}$$

Jawaban: B

35. Lingkaran dengan pusat $(3, -2)$ dan $r = 4$ adalah

$$(x-3)^2 + (y+2)^2 = 16$$

$$\text{Matriks rotasi } [O, 90^\circ] = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\text{Matriks refleksi terhadap sumbu-} X = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned}\begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \\ \Leftrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \Leftrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -y \\ -x \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\text{Diperoleh } x' &= -y \Rightarrow y = -x' \\ y' &= -x \Rightarrow x = -y'\end{aligned}$$

Sehingga persamaan bayangannya adalah

$$\begin{aligned}(-y' - 3)^2 + (-x' + 2)^2 &= 16 \\ \Leftrightarrow y'^2 + 6y' + 9 + x'^2 - 4x' + 4 - 16 &= 0 \\ \Leftrightarrow x'^2 + y'^2 - 4x' + 6y' - 3 &= 0 \\ \Leftrightarrow x^2 + y^2 - 4x + 6y - 3 &= 0\end{aligned}$$

Jawaban: A

36. Data dengan frekuensi terbanyak terdapat di kelas dengan interval 50–59

$$T_b = 50 - 0,5 = 49,5$$

$$l = 10$$

$$s_1 = 9 - 7 = 2$$

$$s_2 = 9 - 6 = 3$$

$$\begin{aligned}Mo &= T_b + \frac{s_1}{s_1 + s_2} \cdot l = 49,5 + \frac{2}{2+3} \cdot 10 \\ &= 49,5 + \frac{2}{5} \cdot 10 = 49,5 + 4 = 53,5\end{aligned}$$

Jawaban: E

37. $n = \sum f_i = 40$

Kuartil tengah = Q_2

Kelas Q_2 berada pada data ke-

$$\frac{2}{4} \times n = \frac{1}{4} \times 40 = 10 \text{ (kelas ke-3)}$$

$$T_b = 51 - 0,5 = 50,5$$

$$l = 10$$

$$\begin{aligned}Q_2 &= T_b + \left(\frac{\frac{2}{4}n - F_k}{f_{Q_2}} \right) l = 50,5 + \left(\frac{20 - 14}{15} \right) 10 \\ &= 50,5 + \frac{6}{15} \cdot 10 = 50,5 + 4 = 54,5\end{aligned}$$

Jawaban: D

38. Bilangan genap yang terdiri atas tiga angka.

Kasus I: Angka satuan genap

Angka satuan terdapat 2 angka

Angka ratusan terdapat 5 angka

Angka puluhan terdapat 5 angka

Banyak bilangan:

$$2 \times 5 \times 5 = 50$$

Kasus II: Angka satuan angka nol

Angka satuan terdapat 1 angka

Angka ratusan terdapat 6 angka

Angka puluhan terdapat 5 angka

Banyak bilangan:

$$1 \times 6 \times 5 = 30$$

Banyak bilangan keseluruhan:

$$50 + 30 = 80$$

Jadi, banyak bilangan genap yang terdiri atas tiga angka berbeda adalah 80.

Jawaban: B

39. Banyak soal bernomor 1, 5, 8, 9, dan 10 ada 5, sehingga 5 soal harus dikerjakan, 1 soal sisanya harus dipilih dari 5 soal yang masih tersedia.

$${}_1C_5 = \frac{5!}{(5-1)!1!} = \frac{5 \cdot 4!}{1 \cdot 4!} = 5$$

Jadi, banyak cara siswa mengerjakan soal sisa adalah 5.

Jawaban: A

40. Pada percobaan lempar undi 2 dadu, $n(S) = 36$

A = Muncul mata dadu berjumlah 7
 $= \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}, n(A) = 6$

B = Muncul mata dadu berjumlah 10
 $= \{(4, 6), (5, 5), (6, 4)\}, n(B) = 3$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) \\ &= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} \\ &= \frac{6}{36} + \frac{3}{36} = \frac{9}{36} \end{aligned}$$

Jawaban: D

PEMBAHASAN DAN KUNCI JAWABAN PEMANTAPAN PAKET 4

MATEMATIKA IPA

$$\begin{aligned} 1. \quad \frac{\left(3 + \left(\frac{1}{5}\right)^{-1}\right)^{-1} (3^{-2} - 5^{-2})}{(3^{-1} + 5^{-1})(3(5^{-1}) - (3^{-1})5)} &= \frac{(3+5)^{-1} \left(\frac{1}{9} - \frac{1}{25}\right)}{\left(\frac{1}{3} + \frac{1}{5}\right) \left(\frac{3}{5} - \frac{5}{3}\right)} \\ &= \frac{\frac{1}{8} \left(\frac{1}{3} - \frac{1}{5}\right)}{\left(\frac{3}{5} - \frac{5}{3}\right)} = \frac{\frac{1}{8} \left(\frac{2}{15}\right)}{-\frac{16}{15}} = -\frac{1}{64} \end{aligned}$$

Jawaban: A

$$\begin{aligned} 2. \quad x^p &= \frac{\sqrt{x} \sqrt{x} \sqrt{x}}{x} = \frac{\sqrt{x} \sqrt{x^{\frac{3}{2}}}}{x} = \frac{\sqrt{x \cdot x^{\frac{3}{2}}}}{x} = \frac{\sqrt{x^{\frac{7}{2}}}}{x} = \frac{x^{\frac{7}{4}}}{x} = x^{-\frac{1}{4}} \\ \text{Jadi, } p &= -\frac{1}{4}. \end{aligned}$$

Jawaban: A

$$\begin{aligned} 3. \quad {}^4\log 6 &= m + 1 \Leftrightarrow {}^{2^2}\log 6 = m + 1 \\ \Leftrightarrow \frac{1}{2} {}^2\log 6 &= m + 1 \Leftrightarrow {}^2\log 6 = 2m + 2 \\ \Leftrightarrow {}^2\log (2 \cdot 3) &= 2m + 2 \Leftrightarrow {}^2\log 2 + {}^2\log 3 = 2m + 2 \\ \Leftrightarrow 1 + {}^2\log 3 &= 2m + 2 \Leftrightarrow {}^2\log 3 = 2m + 1 \\ {}^9\log 8 &= {}^{3^2}\log 2^3 \\ &= \frac{3}{2} {}^3\log 2 = \frac{3}{2^2\log 3} = \frac{3}{2(2m+1)} = \frac{3}{4m+2} \end{aligned}$$

Jawaban: B

$$\begin{aligned} 4. \quad \left(\frac{1}{3}\right)^{2x+1} &> \sqrt{\frac{27}{3^{x-1}}} \\ (3^{-1})^{2x+1} &> (3^{3-(x-1)})^{\frac{1}{2}} \\ 3^{-2x-1} &> 3^{\frac{-x+4}{2}} \\ -2x-1 &> \frac{-x+4}{2} \quad (\text{tanda pertidaksamaan berubah karena dibagi bilangan negatif}) \\ -4x-2 &> -x+4 \\ -3x &> 6 \\ x &< -2 \end{aligned}$$

Jawaban: D

5. Diketahui $f(x) = x^2 - 2$ dan $(f \circ g)(x) = x^2 + 6x + 7$.

$$(f \circ g)(x) = x^2 + 6x + 7$$

$$f(g(x)) = x^2 + 6x + 7$$

$$(g(x))^2 - 2 = x^2 + 6x + 7$$

$$(g(x))^2 = x^2 + 6x + 9$$

$$(g(x))^2 = (x+3)^2$$

$$g(x) = x+3$$

Jawaban: E

6. $f(x) = ax + 3$

$$\text{Misalkan } y = ax + 3 \Leftrightarrow x = \frac{y-3}{a}$$

$$\text{Jadi, } f^{-1}(x) = \frac{y-3}{a}$$

$$f^{-1}(9) = \frac{9-3}{a} = \frac{6}{a}$$

$$f^{-1}(f^{-1}(9)) = 3 \Leftrightarrow f^{-1}\left(\frac{6}{a}\right) = 3 \Leftrightarrow \frac{\frac{6}{a}-3}{a} = 3$$

$$\Leftrightarrow \frac{6}{a} - 3 = 3a$$

Kedua ruas dikali a , diperoleh

$$3a^2 + 3a - 6 = 0 \Leftrightarrow a^2 + a - 2 = 0$$

$$a^2 + a + 1 = (a^2 + a - 2) + 3 = 0 + 3 = 3$$

Jawaban: E

7. $2x^2 - 6x - p = 0$ mempunyai akar-akar x_1 dan x_2 , dengan:

$$x_1 + x_2 = -\frac{-6}{2} = 3 \Leftrightarrow x_1 = 3 - x_2$$

$$x_1 x_2 = \frac{-p}{2}$$

$$\begin{aligned}x_1 - x_2 &= 5 \\3 - x_2 - x_2 &= 5 \\-2x_2 &= 2 \\x_2 &= -1\end{aligned}$$

Substitusikan $x_2 = -1$ ke persamaan $x_1 = 3 - x_2$, diperoleh:

$$\begin{aligned}x_1 &= 3 - x_2 \\x_1 &= 3 - (-1) \\x_1 &= 3 + 1 = 4\end{aligned}$$

Substitusikan $x_1 = 4$ dan $x_2 = -1$ ke persamaan

$$x_1 x_2 = \frac{-p}{2}, \text{ diperoleh:}$$

$$\begin{aligned}x_1 x_2 &= \frac{-p}{2} \\(4)(-1) &= \frac{-p}{2} \\-4 &= \frac{-p}{2} \\-8 &= -p \\p &= 8\end{aligned}$$

Jawaban: E

8. $x^2 - ax + 1 = 0$, akar-akarnya x_1 dan x_2 , sehingga

$$x_1 + x_2 = a \text{ dan } x_1 x_2 = 1$$

$$x^2 + px + q = 0, \text{ akar-akarnya } \frac{x_1^2}{x_2} \text{ dan } \frac{x_2^2}{x_1}, \text{ sehingga}$$

$$\begin{aligned}\frac{x_1^2}{x_2} + \frac{x_2^2}{x_1} &= p \\ \frac{x_1^3 + x_2^3}{x_1 x_2} &= p \\ \frac{(x_1 + x_2)^3 - 3x_1 x_2 (x_1 + x_2)}{x_1 x_2} &= p \\ \frac{a^3 - 3(1)(a)}{1} &= p \\ p &= a^3 - 3a\end{aligned}$$

Jawaban: B

9. Pernyataan "Jika semua siswa rajin belajar, maka Dewa naik kelas"

Pernyataan di atas merupakan implikasi yang mengandung pernyataan berkuantor (semua)

$$\text{Ingat: } (p \Rightarrow q) \equiv (\sim p \vee q)$$

$$\sim(\text{semua } p) \equiv \text{ada } \sim p$$

Sehingga pernyataan yang ekuivalen dengan pernyataan di atas adalah:

"Ada siswa tidak rajin belajar atau Dewa naik kelas"

Jawaban: D

10. $f(1) = 0 \Leftrightarrow$ karena $f(x) \geq 0$, maka titik $(1, 0)$ adalah titik balik minimum, sehingga

$$f(x) = a(x - x_p)^2 + y_p$$

$$\Leftrightarrow f(x) = a(x - 1)^2$$

$$f(2) = 2 \Leftrightarrow a(2 - 1)^2 = 2 \Leftrightarrow a = 2$$

sehingga

$$f(x) = 2(x - 1)^2$$

$$\begin{aligned}f(0) + f(4) &= 2(0 - 1)^2 + 2(4 - 1)^2 \\&= 2 + 18 = 20\end{aligned}$$

Jawaban: B

$$11. \frac{x}{2} + \frac{y}{4} = 6 \text{ (dikali 4)} \Rightarrow 2x + y = 24 \quad \dots (1)$$

$$\frac{y}{6} - \frac{z}{2} = -2 \text{ (dikali 6)} \Rightarrow y - 3z = -12 \quad \dots (2)$$

$$\frac{z}{4} + \frac{x}{3} = 4 \text{ (dikali 12)} \Rightarrow 3z + 4x = 48 \quad \dots (3)$$

$$(1) + (2) + (3) \Rightarrow 6x + 2y = 60 \Leftrightarrow 3x + y = 30 \quad \dots (4)$$

$$(4) - (1) \Rightarrow x = 6$$

Substitusikan $x = 6$ ke (1) diperoleh

$$2(6) + y = 24 \Leftrightarrow y = 12$$

Substitusikan $x = 6$ ke (3) diperoleh

$$3z + 4(6) = 48 \Leftrightarrow z = 8$$

$$\text{Jadi, } x + y + z = 6 + 12 + 8 = 26$$

Jawaban: E

12. Persamaan garis melalui $(4, 0)$ dan $(0, 8)$

$$\Rightarrow 8x + 4y = 32 \Leftrightarrow 2x + y = 8$$

Persamaan garis melalui $(9, 0)$ dan $(0, 6)$

$$\Rightarrow 6x + 9y = 54 \Leftrightarrow 2x + 3y = 18$$

Persamaan garis melalui $(12, 0)$ dan $(0, 4)$

$$\Rightarrow 4x + 12y = 48 \Leftrightarrow x + 3y = 12$$

Titik potong garis $2x + y = 8$ dan $2x + 3y = 18$ adalah

$$\left(\frac{3}{2}, 5\right).$$

Titik potong garis $x + 3y = 12$ dan $2x + 3y = 18$ adalah $(6, 2)$

$$f(x, y) = 500x + 1.000y$$

$$(12, 0) \Rightarrow f = 500(12) + 1.000(0) = 6.000$$

$$(0, 8) \Rightarrow f = 500(0) + 1.000(8) = 8.000$$

$$(6, 2) \Rightarrow f = 500(6) + 1.000(2) = 5.000 \text{ (minimum)}$$

$$\left(\frac{3}{2}, 5\right) \Rightarrow f = 500\left(\frac{3}{2}\right) + 1.000(5) = 5.750$$

Jawaban: D

$$13. \begin{pmatrix} -1 & d \\ -b & 3 \end{pmatrix} + \begin{pmatrix} 4 & -5 \\ -3 & b \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 2c & 1 \\ c & a+1 \end{pmatrix}$$

$$\begin{pmatrix} -1+4 & d-5 \\ -b-3 & 3+b \end{pmatrix} = \begin{pmatrix} 4c-c & 2-a-1 \\ -8c+3c & -4+3a+3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & d-5 \\ -b-3 & b+3 \end{pmatrix} = \begin{pmatrix} 3c & 1-a \\ -5c & 3a-1 \end{pmatrix}$$

Diperoleh:

- $3 = 3c \Leftrightarrow c = 1$
- $-b - 3 = -5c$
 $-b - 3 = -5(1)$
 $-b - 3 = -5$
 $-b = -2$
 $b = 2$
- $b + 3 = 3a - 1$
 $2 + 3 = 3a - 1$
 $5 = 3a - 1$
 $6 = 3a$
 $a = 2$

Sehingga,

$$a + c = 2 + 1 = 3$$

Jawaban: B

$$14. \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} 2a & 3b \\ -2 & c \end{pmatrix} + \begin{pmatrix} b & 2c \\ d & -4 \end{pmatrix}$$

$$\begin{pmatrix} -1+4 & 3-10 \\ -4+6 & 12-15 \end{pmatrix} = \begin{pmatrix} 2a+b & 3b+2c \\ -2+d & c-4 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -7 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 2a+b & 3b+2c \\ d-2 & c-4 \end{pmatrix}$$

Diperoleh:

- $-3 = c - 4 \Leftrightarrow c = 1$
- $2 = d - 2 \Leftrightarrow d = 4$
- $-7 = 3b + 2c$
 $-7 = 3b + 2(1)$
 $-7 = 3b + 2$
 $-9 = 3b$
 $b = -3$
- $3 = 2a + b$
 $3 = 2a + (-3)$
 $3 = 2a - 3$
 $6 = 2a$
 $a = 3$

Sehingga,

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ -3 & 4 \end{pmatrix}$$

$$\begin{vmatrix} 3 & 1 \\ -3 & 4 \end{vmatrix} = \frac{1}{3 \cdot 4 - (-3) \cdot 1} \begin{pmatrix} 4 & -1 \\ 3 & 3 \end{pmatrix}$$

$$= \frac{1}{12+3} \begin{pmatrix} 4 & -1 \\ 3 & 3 \end{pmatrix}$$

$$= \frac{1}{15} \begin{pmatrix} 4 & -1 \\ 3 & 3 \end{pmatrix}$$

Jawaban: E

15. Bilangan ganjil positif yang lebih kecil dari 200 serta habis dibagi 9 adalah 9, 27, 45, 63, ..., 189

Barisan ini merupakan barisan aritmetika dengan $a = 9$ dan $b = 18$.

$$U_n = a + b(n - 1)$$

$$189 = 9 + 18(n - 1) \Rightarrow n = 11$$

$$S_n = \frac{n}{2}(a + U_n)$$

$$S_{11} = \frac{11}{2}(9 + 189) = 1.089$$

Jawaban: A

$$16. U_1 + U_2 + U_3 + \dots = 3 \Leftrightarrow a + ar + ar^2 + ar^3 + \dots = 3$$

$$S = \frac{a}{1-r} \Leftrightarrow \frac{a}{1-r} = 3 \Leftrightarrow a = 3(1-r) \quad \dots (1)$$

$$U_3 + U_4 + U_5 + \dots = 1 \Leftrightarrow ar^2 + ar^3 + ar^4 + \dots = 1$$

$$S = \frac{ar^2}{1-r} \Leftrightarrow \frac{ar^2}{1-r} = 1 \quad \dots (2)$$

Substitusikan (1) ke (2) diperoleh

$$\frac{3(1-r)r^2}{1-r} = 1 \Leftrightarrow 3r^2 = 1 \Leftrightarrow r^2 = \frac{1}{3} \Leftrightarrow r = \pm \frac{1}{\sqrt{3}}$$

Jawaban: D

17. Awal pengamatan $\Rightarrow x$

Setelah 6 jam $\Rightarrow 2x$

$$\text{Setelah 12 jam} \Rightarrow 4x - \frac{1}{4}(4x) = 3x$$

Setelah 18 jam $\Rightarrow 6x$

$$\text{Setelah 24 jam} \Rightarrow 12x - \frac{1}{4}(12x) = 9x$$

Setelah 30 jam $\Rightarrow 18x$

$$\text{Setelah 36 jam} \Rightarrow 36x - \frac{1}{4}(36x) = 27x$$

Jawaban: C

$$18. x^2 - 3x + 2 = 0 \Leftrightarrow x^2 = 3x - 2$$

	a	0	b	$a + b$
3	$*$	$3a$	$9a$	$*$
-2	$*$	$*$	$-2a$	$-6a$
	a	$3a$	$7a + b$	$-5a + b$

$$\text{Sisa} \Rightarrow (7a + b)x + (-5a + b) = x + 1$$

Dengan kesamaan koefisiennya, diperoleh

$$7a + b = 1 \quad \dots (1)$$

$$-5a + b = 1 \quad \dots (2)$$

Dari (1) dan (2) diperoleh $a = 0$ dan $b = 1$

$$a - b = 0 - 1 = -1$$

Jawaban: E

$$\begin{aligned} 19. \lim_{x \rightarrow 2} \frac{\sqrt{5x-1} - \sqrt{6x-3}}{x-2} &= \lim_{x \rightarrow 2} \frac{\sqrt{5x-1} - \sqrt{6x-3}}{x-2} \cdot \frac{\sqrt{5x-1} + \sqrt{6x-3}}{\sqrt{5x-1} + \sqrt{6x-3}} \\ &= \lim_{x \rightarrow 2} \frac{(5x-1) - (6x-3)}{(x-2)(\sqrt{5x-1} + \sqrt{6x-3})} \\ &= \lim_{x \rightarrow 2} \frac{-x+2}{(x-2)(\sqrt{5x-1} + \sqrt{6x-3})} \\ &= \lim_{x \rightarrow 2} \frac{-1}{\sqrt{5x-1} + \sqrt{6x-3}} \\ &= \frac{-1}{\sqrt{5 \cdot 2 - 1} + \sqrt{6 \cdot 2 - 3}} \\ &= \frac{-1}{\sqrt{9} + \sqrt{9}} = \frac{-1}{3+3} = \frac{-1}{6} \end{aligned}$$

Jawaban: A

$$\begin{aligned} 20. \lim_{x \rightarrow \infty} (\sqrt{4x^2 + x - 3} + 5 - 2x) &= \lim_{x \rightarrow \infty} (\sqrt{4x^2 + x - 3} - 2x) + 5 \\ &= \lim_{x \rightarrow \infty} (\sqrt{4x^2 + x - 3} - \sqrt{4x^2}) + 5 \\ &= \frac{1-0}{2\sqrt{4}} + 5 = \frac{1}{4} + 5 = 5\frac{1}{4} \end{aligned}$$

Jawaban: E

$$21. f(x) = x^3 - \frac{3}{2}x^2 - 6x + 7$$

$$f'(x) = 3x^2 - 3x - 6$$

$$f''(x) = 6x - 3$$

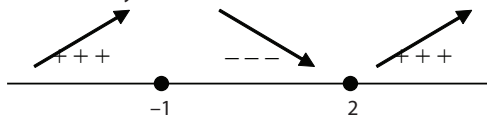
Nilai stasioner $f'(x) = 0$

$$3x^2 - 3x - 6 = 0$$

$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0$$

Nilai batasnya $x = -1$ dan $x = 2$



Berdasarkan garis bilangan di atas dapat diperoleh fungsi f mencapai nilai minimum untuk $x = 2$ dan maksimum untuk $x = -1$.

Sehingga,

$$\begin{aligned} f(-1) &= (-1)^3 - \frac{3}{2}(-1)^2 - 6(-1) + 7 \\ &= -1 - \frac{3}{2} + 6 + 7 = 10\frac{1}{2} \end{aligned}$$

$$\begin{aligned} f(2) &= (2)^3 - \frac{3}{2}(2)^2 - 6(2) + 7 \\ &= 8 - 6 - 12 + 7 = -3 \end{aligned}$$

Jawaban: D

$$22. \text{ Misalkan } u = 3x^2 + 3x - 5$$

$$\frac{du}{dx} = 6x + 3 = 3(2x + 1)$$

$$dx = \frac{du}{3(2x+1)}$$

Sehingga,

$$\begin{aligned} \int \frac{2x+1}{\sqrt{3x^2+3x-5}} dx &= \int (2x+1) u^{-\frac{1}{2}} \frac{du}{3(2x+1)} \\ &= \frac{1}{3} \int u^{-\frac{1}{2}} du \\ &= \frac{1}{3} \cdot \left(2u^{\frac{1}{2}} \right) + C = \frac{2}{3} \sqrt{u} + C \\ &= \frac{2}{3} \sqrt{3x^2+3x-5} + C \end{aligned}$$

Jawaban: B

$$\begin{aligned} 23. \int_1^3 (x^2 - x - 2) dx &= \left(\frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x \right) \Big|_1^3 \\ &= \frac{1}{3}(3^3 - 1^3) - \frac{1}{2}(3^2 - 1^2) - 2(3 - 1) \\ &= \frac{1}{3}(27 - 1) - \frac{1}{2}(9 - 1) - 2(3 - 1) \\ &= \frac{26}{3} - \frac{8}{2} - 4 = 8\frac{2}{3} - 4 - 4 = \frac{2}{3} \end{aligned}$$

Jawaban: D

$$24. p(x) = 90x - 3x^2$$

$$p'(x) = 90 - 6x$$

Nilai maksimum jika dan hanya jika $p'(x) = 0$

$$p'(x) = 0$$

$$90 - 6x = 0$$

$$6x = 90$$

$$x = 15$$

$$\begin{aligned} p(15) &= 90(15) - 3(15)^2 \\ &= 1.350 - 675 = 675 \end{aligned}$$

Jadi, hasil penjualan maksimum yang diperoleh adalah Rp675.000,00.

Jawaban: D

25. Diketahui: $\cos A \cos B = \frac{1}{2}$. Karena $\cos A \cos B$ ada nilainya, maka sudut C pasti siku-siku ($\cos 90^\circ = 0$).

Pada segitiga berlaku $A + B + C = 180^\circ \Leftrightarrow A + B = 90^\circ$

$$\Leftrightarrow \cos(A + B) = \cos 90^\circ$$

$$\Leftrightarrow \cos A \cos B - \sin A \sin B = 0$$

$$\Leftrightarrow \frac{1}{2} - \sin A \sin B = 0 \Leftrightarrow \sin A \sin B = \frac{1}{2}$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B = \frac{1}{2} + \frac{1}{2} = 1$$

Jawaban: A

$$\begin{aligned} 26. \frac{\cos 280^\circ + \cos 140^\circ}{\sin 280^\circ + \sin 140^\circ} &= \frac{2 \cos \left(\frac{280^\circ + 140^\circ}{2} \right) \cos \left(\frac{280^\circ - 140^\circ}{2} \right)}{2 \sin \left(\frac{280^\circ + 140^\circ}{2} \right) \cos \left(\frac{280^\circ - 140^\circ}{2} \right)} \\ &= \frac{2 \cos 210^\circ \cos 70^\circ}{2 \sin 210^\circ \cos 70^\circ} \\ &= \frac{\cos 210^\circ}{\sin 210^\circ} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3} \end{aligned}$$

Jawaban: A

27. Jika diperhatikan grafik pada soal adalah grafik fungsi kosinus, bentuk umum fungsi sinus adalah $f(x) = y = A \cos k(x + b)$

Amplitudo titik puncak, $A = 3$

Periode grafik, $p = \pi$

$$k = \frac{2\pi}{p} = \frac{2\pi}{\pi} = 2$$

Persamaan fungsinya adalah

$$y = 3 \cos 2(x + b)$$

Grafik fungsi melalui titik $\left(\frac{\pi}{4}, 0\right)$

$$0 = 3 \cos 2\left(\frac{\pi}{4} + b\right)$$

$$0 = \cos 2\left(\frac{\pi}{4} + b\right)$$

$$\cos \frac{\pi}{2} = \cos 2\left(\frac{\pi}{4} + b\right)$$

$$\frac{\pi}{2} = \frac{\pi}{2} + 2b$$

$$b = 0$$

Jadi, persamaan fungsinya adalah

$$y = 3 \cos 2(x + 0) \text{ atau}$$

$$y = 3 \cos 2x$$

Jawaban: A

28. $\cos 2x - 5 \cos x = 2; 0^\circ \leq x \leq 360^\circ$

$$(2 \cos^2 x - 1) - 5 \cos x - 2 = 0$$

$$2 \cos^2 x - 5 \cos x - 3 = 0$$

$$(2 \cos x + 1)(\cos x - 3) = 0$$

$$\cos x = -\frac{1}{2} \text{ atau } \cos x = 3 \text{ (tidak memenuhi)}$$

$$x = 120^\circ, 240^\circ$$

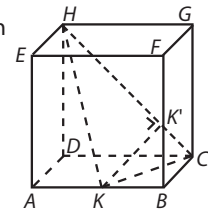
Jadi, himpunan penyelesaiannya $\{120^\circ, 240^\circ\}$.

Jawaban: C

29. $HC = \text{diagonal bidang} = 12\sqrt{2} \text{ cm}$

Perhatikan $\triangle KBC$.

$$\begin{aligned} KC &= \sqrt{KB^2 + BC^2} \\ &= \sqrt{6^2 + 12^2} = \sqrt{36 + 144} \\ &= \sqrt{180} = 6\sqrt{5} \text{ cm} \end{aligned}$$



Perhatikan $\triangle HKD$.

$$DK = CK = 6\sqrt{5}$$

$$\begin{aligned} HK &= \sqrt{DK^2 + HD^2} = \sqrt{(6\sqrt{5})^2 + 12^2} \\ &= \sqrt{180 + 144} = \sqrt{324} = 18 \end{aligned}$$

Karena $\triangle KHC$ sembarang, maka

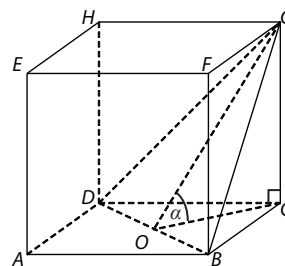
$$\begin{aligned} CK' &= \frac{CK^2 + CH^2 - KH^2}{2CH} = \frac{(6\sqrt{5})^2 + (12\sqrt{2})^2 - 18^2}{2(12\sqrt{2})} \\ &= \frac{180 + 288 - 324}{24\sqrt{2}} = \frac{144}{24\sqrt{2}} = \frac{144}{24\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} KK' &= \sqrt{KC^2 - (CK')^2} \\ &= \sqrt{(6\sqrt{5})^2 - (3\sqrt{2})^2} = \sqrt{162} = 9\sqrt{2} \end{aligned}$$

Jadi, jarak K ke garis HC adalah $9\sqrt{2} \text{ cm}$.

Jawaban: D

30. Perhatikan kubus $ABCD.EFGH$ di bawah ini.

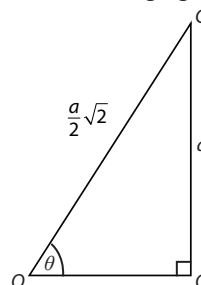


Misalkan panjang rusuknya a .

$$\angle(BDG, ABCD) = \angle(GO, OC) = \theta$$

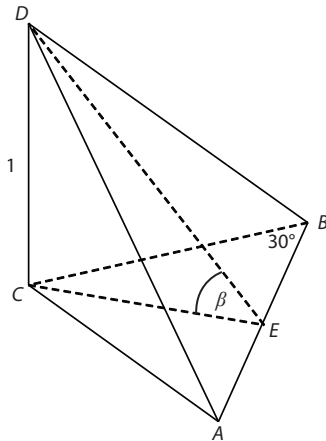
$$OC = \frac{1}{2} AC = \frac{1}{2} (a\sqrt{2}) = \frac{a}{2}\sqrt{2}$$

Perhatikan segitiga CGO siku-siku di O .



$$\begin{aligned}
 GO &= \sqrt{GC^2 + CO^2} = \sqrt{a^2 + \left(\frac{a}{2}\sqrt{2}\right)^2} \\
 &= \sqrt{a^2 + \frac{a^2}{2}} = \sqrt{\frac{3a^2}{2}} = \frac{a}{2}\sqrt{6} \\
 \sin \theta &= \frac{GC}{GO} = \frac{a}{\frac{a}{2}\sqrt{6}} = \frac{2}{\sqrt{6}} = \frac{1}{3}\sqrt{6}
 \end{aligned}$$

31. Perhatikan $D.ABC$ di bawah ini.



$$CB = \frac{1}{\tan 30^\circ} = \frac{1}{\frac{1}{\sqrt{3}}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\begin{aligned}
 DB &= \sqrt{CB^2 + DC^2} = \sqrt{(\sqrt{3})^2 + 1^2} \\
 &= \sqrt{3+1} = \sqrt{4} = 2
 \end{aligned}$$

$$AB = AC = CB = \sqrt{3}$$

$$DA = DB = 2$$

$$\angle(CAB, DAB) = \angle(CE, DE) = \beta$$

Perhatikan segitiga DAB .

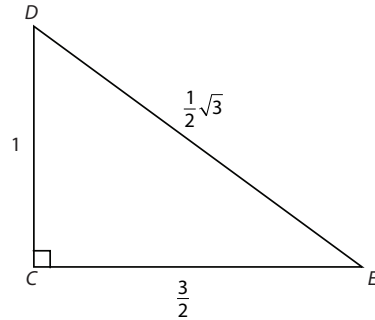
$$\begin{aligned}
 DE &= \sqrt{DB^2 - BE^2} = \sqrt{2^2 - \left(\frac{\sqrt{3}}{2}\right)^2} \\
 &= \sqrt{4 - \frac{3}{4}} = \sqrt{\frac{13}{4}} = \frac{1}{2}\sqrt{13}
 \end{aligned}$$

Perhatikan segitiga ABC .

$$\begin{aligned}
 CE &= \sqrt{CB^2 - EB^2} = \sqrt{(\sqrt{3})^2 - \left(\frac{\sqrt{3}}{2}\right)^2} \\
 &= \sqrt{3 - \frac{3}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2}
 \end{aligned}$$

Jawaban: B

Perhatikan segitiga DEC siku-siku di C .



$$\cos \beta = \frac{CE}{DE} = \frac{\frac{3}{2}}{\frac{1}{2}\sqrt{13}} = \frac{3}{\sqrt{13}} = \frac{3}{13}\sqrt{13}$$

Jawaban: B

$$\begin{aligned}
 32. \quad \vec{a} &= \overrightarrow{AB} = B - A = (1, 1, 0) \Rightarrow |\vec{a}| = \sqrt{1^2 + 1^2 + 0} = \sqrt{2} \\
 \vec{b} &= \overrightarrow{AC} = C - A = (0, 1, 1) \Rightarrow |\vec{b}| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2} \\
 \vec{a} \cdot \vec{b} &= 1(0) + 1(1) + 0(1) = 1 \\
 \cos \angle(\vec{a}, \vec{b}) &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{1}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2} \\
 \angle(\vec{a}, \vec{b}) &= 60^\circ
 \end{aligned}$$

Jawaban: C

33. Pusat $(2, -1)$ dan menyinggung sumbu- Y , maka $r = 2$.

Persamaan lingkaran:

$$(x - 2)^2 + (y + 1)^2 = 2^2$$

$$x^2 - 4x + 4 + y^2 + 2y + 1 = 4$$

$$x^2 + y^2 - 4x + 2y + 1 = 0$$

Jawaban: C

$$34. \quad x^2 + y^2 - 6x + 4y + 9 = 0$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 = 4$$

$$(x - 3)^2 + (y + 2)^2 = 2^2$$

Diperoleh pusat $(3, -2)$ dan $r = 2$

Gradien dari garis $y - 3x - 1 = 0$ adalah $m = 3$. Karena sejajar, maka $m_1 = m = 3$.

Persamaan garis singgung:

$$y - y_p = m(x - x_p) \pm r\sqrt{1 + m^2}$$

$$y + 2 = 3(x - 3) \pm 2\sqrt{1 + 3^2}$$

$$y + 2 = 3x - 9 \pm 2\sqrt{1 + 9}$$

$$y = 3x - 11 \pm 2\sqrt{10} \Leftrightarrow \begin{cases} 3x - y - 11 + 2\sqrt{10} = 0 \\ 3x - y - 11 - 2\sqrt{10} = 0 \end{cases}$$

Jawaban: B

35. Refleksi terhadap sumbu-X, $T_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Refleksi terhadap garis $y = x$, $T_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Refleksi terhadap sumbu-X dilanjutkan dengan refleksi terhadap garis $y = x$ adalah:

$$T_2 \circ T_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

sehingga $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}$

$$x' = -y \Rightarrow y = -x'$$

$$y' = x \Rightarrow x = y'$$

$$y = 3x + 3 \xrightarrow{T_2 \circ T_1} -x' = 3y' + 3 \Leftrightarrow -x = 3y + 3$$

$$3y = -x - 3 \Leftrightarrow y = -\frac{1}{3}x - 1$$

Jawaban: E

36. $n = \sum f_i = 80$

Median = Me

Kelas Me berada pada data ke-

$$\frac{1}{2} \times n = \frac{1}{2} \times 80 = 40 \text{ (kelas ke-4)}$$

$$T_b = 45,5$$

$$l = 5$$

$$\begin{aligned} Me &= T_b + \left(\frac{\frac{1}{2}n - F_k}{f_{Me}} \right) l \\ &= 45,5 + \left(\frac{40 - 37}{20} \right) 5 \\ &= 45,5 + \frac{3}{20} \cdot 5 \\ &= 45,5 + 0,75 = 46,25 \end{aligned}$$

Jawaban: B

37. $n = \sum f_i = 24$

Kuartil atas = Q_3

Kelas Q_3 berada pada data ke-

$$\frac{3}{4} \times n = \frac{3}{4} \times 24 = 18 \text{ (kelas ke-4)}$$

$$T_b = 56 - 0,5 = 55,5$$

$$l = 3$$

$$\begin{aligned} Q_3 &= T_b + \left(\frac{\frac{3}{4}n - F_k}{f_{Q_3}} \right) l \\ &= 55,5 + \left(\frac{18 - 17}{6} \right) 3 \\ &= 55,5 + \frac{1}{6} \cdot 3 \\ &= 55,5 + 0,5 = 56 \end{aligned}$$

Jawaban: B

38. Bilangan kurang dari 500 yang terdiri atas tiga angka berbeda.

Angka ratusan terdapat 3 angka

Angka puluhan terdapat 6 angka

Angka satuan terdapat 5 angka

Banyak bilangan:

$$3 \times 6 \times 5 = 90$$

Jadi, banyak bilangan kurang dari 500 yang terdiri atas tiga angka berbeda adalah 90.

Jawaban: D

39. Banyak soal bernomor ganjil yaitu 1, 3, 5, 7, 9, dan 11 ada 6, sehingga 6 soal harus dikerjakan, 4 soal sisanya harus dipilih dari 6 soal yang masih tersedia.

$${}_4C_6 = \frac{6!}{(6-4)!4!} = \frac{6 \times 5 \times 4!}{2 \times 1 \times 4!} = 3 \times 5 = 15$$

Jawaban: B

40. Peluang terambilnya As = $P(\text{As}) = \frac{4}{52} = \frac{1}{13}$

$$\text{Peluang terambilnya kartu hitam} = P(\text{Hitam}) = \frac{26}{52} = \frac{1}{2}$$

Peluang terambil kartu As dan Hitam = $P(\text{As} \cap \text{Hitam})$

$$= \frac{2}{52} = \frac{1}{26} \text{ (Ada 2 kartu As yang hitam)}$$

$$P(\text{As} \cup \text{Hitam}) = P(\text{As}) + P(\text{Hitam}) - P(\text{As} \cap \text{Hitam})$$

$$= \frac{1}{13} + \frac{1}{2} - \frac{1}{26} = \frac{7}{13}$$

Jawaban: D

KUNCI JAWABAN PREDIKSI UNBK/UNKP

MATEMATIKA
IPA

- | | | | |
|-------|-------|-------|-----------------|
| 1. E | 11. D | 21. B | 31. D |
| 2.a B | 12. A | 22. D | 32. A |
| 3. A | 13. E | 23. B | 33. C |
| 4. C | 14. B | 24. C | 34. C |
| 5. B | 15. C | 25. E | 35. D |
| 6. A | 16. E | 26. D | 36. D |
| 7. D | 17. D | 27. A | 37. -5 |
| 8. E | 18. C | 28. C | 38. 3 |
| 9. B | 19. B | 29. C | 39. 330° |
| 10. C | 20. A | 30. C | 40. 80 |

KUNCI JAWABAN PREDIKSI USBN

MATEMATIKA
UMUM

A. Pilihan Ganda

- | | | | | |
|------|-------|-------|-------|-------|
| 1. B | 7. E | 13. B | 19. A | 25. C |
| 2. C | 8. B | 14. C | 20. D | 26. A |
| 3. A | 9. A | 15. E | 21. E | 27. A |
| 4. E | 10. D | 16. D | 22. B | 28. B |
| 5. B | 11. A | 17. B | 23. C | 29. B |
| 6. D | 12. E | 18. C | 24. D | 30. B |

B. Uraian

31. $\frac{3}{5}$

32. 1

33. Rp15.000,00

34. a. $30x + 17y \leq 940$

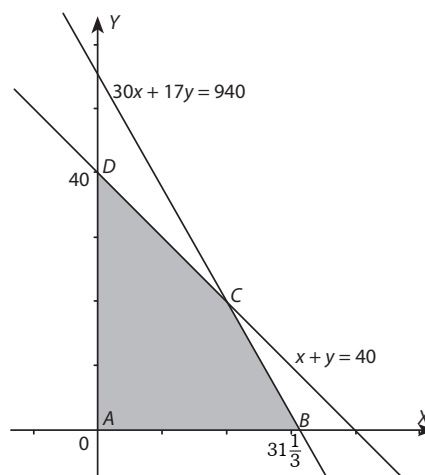
$x + y \leq 40$

$x \geq 0, y \geq 0$

Fungsi objektif:

$f(x, y) = 10.000x + 8.000y$

b.



c. Rp360.000,00

35. 58,75

KUNCI JAWABAN PREDIKSI USBN

MATEMATIKA PEMINATAN

A. Pilihan Ganda

- | | | | | | |
|------|-------|-------|-------|-------|---------------------------|
| 1. D | 7. E | 13. D | 19. B | 25. C | 34. $\frac{1}{3}\sqrt{3}$ |
| 2. B | 8. D | 14. C | 20. E | 26. B | 35. $250\sqrt{3}$ meter |
| 3. E | 9. B | 15. C | 21. A | 27. A | |
| 4. A | 10. A | 16. A | 22. D | 28. C | |
| 5. E | 11. C | 17. E | 23. E | 29. C | |
| 6. C | 12. B | 18. B | 24. C | 30. D | |

B. Uraian

31. $\{(-2, 0), (1, 3)\}$
32. $120\frac{2}{5}\pi$ satuan volume
33. 135°